– PANTHÉON - SORBONNE –
UNIVERSITÉ PARIS 1

# Portfolio Choice

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Session 2

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Variances

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# Part 2. The Theory of Choice under Uncertainty

2.1 Five Axioms

2.2 Utility Functions

2.3 Risk Aversion and Attitudes Towards Risks

2.4 Stochastic Dominance

2.5 Non-Expected Utility Theory

# Basic ideas

- **How people make choices when faced with uncertainty?**
  - Allocation decisions
  - Various criteria
  - Axioms of behavior
  - Parameterizing the objects of choice (mean, variances, etc.)

- **Other approaches : anthropology, sociology, psychology**

# Which criterion?

- Some famous decision rules

| | |
|---|---|
| - Best guaranteed payoff – maximin | (Wald) |
| - Optimism – pessimism index | (Hurwicz) |
| - Minimise regret | (Savage) |
| - Highest average payoff | (Laplace) |
| - Expected profit maximisation | |
| - ... | |
| - Maximise the expected utility | |

# St. Petersburg Paradox

- Toss coin until you get a head, n tosses, win $2^{(n)}$ coins.
– How much would you pay to play this game?
– Expected payoff or gain of this gamble is infinite:

$$E(G) = \frac{1}{2}2 + \frac{1}{2^2}2^2 + \frac{1}{2^3}2^3 + \ldots = 1 + 1 + 1 + \ldots\ldots = \sum_{n=1}^{+\infty}\left(\frac{1}{2^n}\right)2^n = +\infty$$

- Bernoulli suggests that E[G] ≠ E[U(G)]
- Each additional unit of wealth is worth less than the previous one.

$$EU(G) = \sum_{n=1}^{+\infty}\left(\frac{1}{2^n}\right)\ln 2^n = \ln(4)$$

# 2.1 Five Axioms

# Foundations of the expected utility theory

- Objective: to develop a theory of rational decision-making under uncertainty with the minimum sets of reasonable assumptions possible

- The following <u>**five axioms**</u> of cardinal utility provide the minimum set of conditions for consistent and rational behaviour

- What do these axioms of expected utility mean?

1. all individuals are assumed to make completely rational decisions (reasonable)

2. people are assumed to make these rational decisions among thousands of alternatives (hard)

## 5 Axioms of Choice under uncertainty

A1.**Comparability (also known as completeness)**.

For the entire set of uncertain alternatives, an individual can say either that

either x is preferred to outcome y ($x \succ y$)

or y is preferred to x ($y \succ x$)

or indifferent between x and y ($x \sim y$).

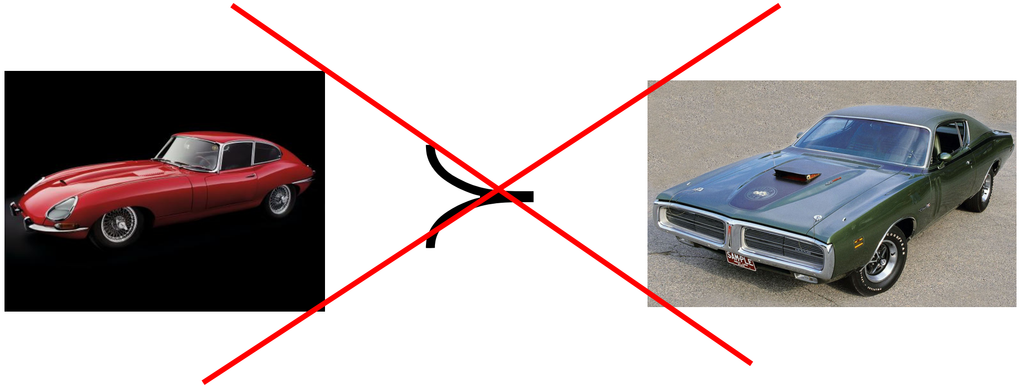
A2.**Transitivity (also know as consistency)**.

If an individual prefers x to y and y to z, then x is preferred to z. If ($x \succ y$ and $y \succ z$, then $x \succ z$).

Similarly, if an individual is indifferent between x and y and is also indifferent between y and z, then the individual is indifferent between x and z. If ($x \sim y$ and $y \sim z$, then $x \sim z$).

# Rational?

 $\succ$  $\succ$ 



A3.**Strong Independence**.

Suppose we construct a gamble where the individual has a probability $\alpha$ of receiving outcome x and a probability $(1-\alpha)$ of receiving outcome z. This gamble is written as:

$$G(x,z:\alpha)$$

Strong independence says that if the individual is indifferent to x and y, then he will also be indifferent as to a first gamble set up between x with probability $\alpha$ and a mutually exclusive outcome z, and a second gamble set up between y with probability $\alpha$ and the same mutually exclusive outcome z.

$$\text{If } x \sim y, \text{ then } G(x,z:\alpha) \sim G(y,z:\alpha)$$

# 5 Axioms of Choice under uncertainty

A4.**Measurability.** (CARDINAL UTILITY)

If outcome y is less preferred than x ($y \prec x$) but more than z ($y \succ z$),

then there is a unique probability $\alpha$ such that:

the individual will be indifferent between

[1] y and

[2] a gamble between x with probability $\alpha$

z with probability $(1-\alpha)$.

In Maths,

if $x \succ y \succeq z$ or $x \succeq y \succ z$,

then there exists a unique $\alpha$ such that $y \sim G(x,z:\alpha)$

# 5 Axioms of Choice under uncertainty

A5.**Ranking**. (CARDINAL UTILITY)

If alternatives y and u both lie somewhere between x and z and we can establish gambles such that an individual is indifferent between y and a gamble between x (with probability $\alpha_1$)and z, while also indifferent between u and a second gamble, this time between x (with probability $\alpha_2$) and z, then if $\alpha_1$ is greater than $\alpha_2$, y is preferred to u.

If $x \succeq y \succeq z$ and $x \succeq u \succeq z$

then if $y \sim G(x,z:\alpha_1)$ and $u \sim G(x,z:\alpha_2)$,
then it follows that if $\alpha_1 \succ \alpha_2$ then $y \succ u$,

or if $\alpha_1 = \alpha_2$, then $y \sim u$

# 2.2 Utility Functions

# From VNM Axioms to Expected Utility Theory

- People are greedy, prefer more wealth than less.

- The 5 axioms and this assumption is all we need in order to develop a expected utility theorem and actually apply the rule of:

$$\max E[U(W)] = \max \sum_i \alpha_i U(W_i)$$

# Utility function properties

- Utility functions must have 2 properties

1. Order preserving: if $U(x) > U(y) \Rightarrow x \succ y$

2. Expected utility can be used to rank combinations of risky alternatives:

$$U[G(x,y:\alpha)] = \alpha U(x) + (1-\alpha) U(y)$$

# Remark

• Utility functions are unique to individuals

- There is no way to compare one individual's utility function with another individual's utility

- Interpersonal comparisons of utility are not possible

if we give 2 people $1,000 there is no way to determine who is happier

# 2.3 Risk Aversion and Attitudes Towards Risks

# Risk aversion

- Consider the following gamble:
- Prospect a: prob = $\alpha$
- Prospect b: prob = $1-\alpha$
- $G(a,b:\alpha)$

- Do you prefer the expected value of the gamble with certainty, or do you prefer the gamble itself?

## Example

- Consider the gamble with

- 10% chance of winning €100

- 90% chance of winning €0

- E(gamble) = €10

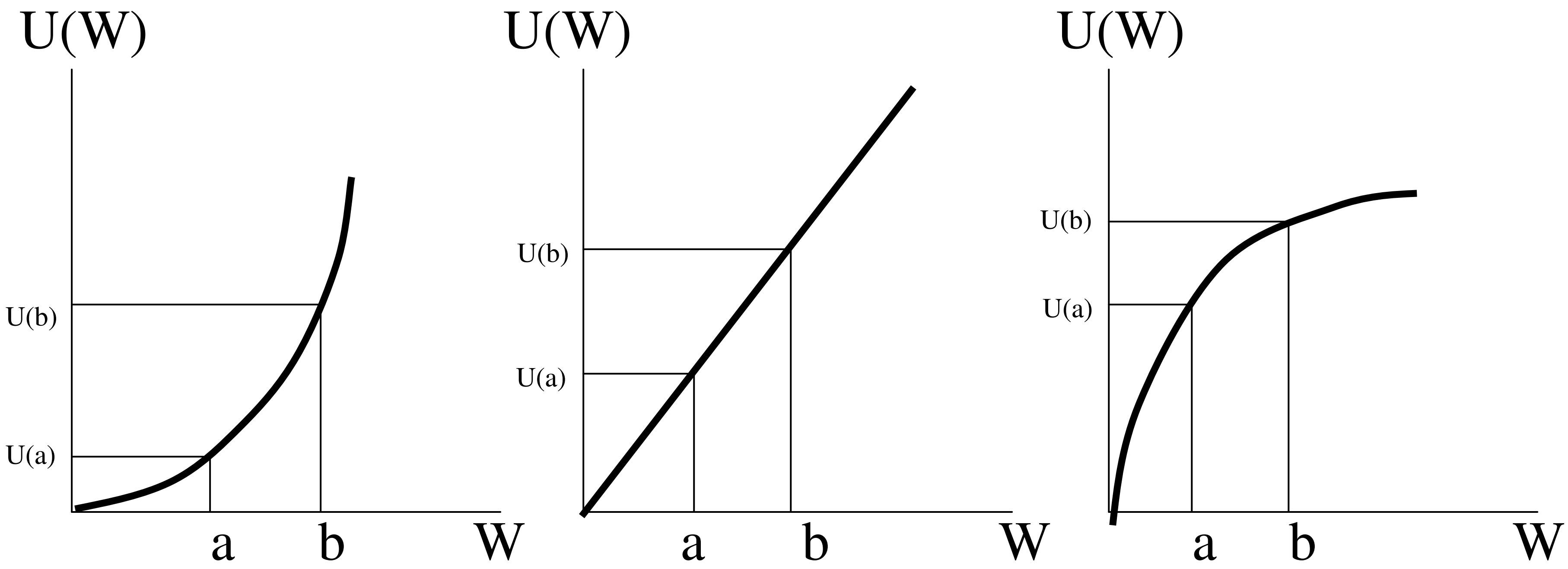
- Do you prefer the €10 for sure or would you prefer the gamble?

- if you prefer the gamble, you are **risk loving**

- if you are indifferent to the options, **risk neutral**

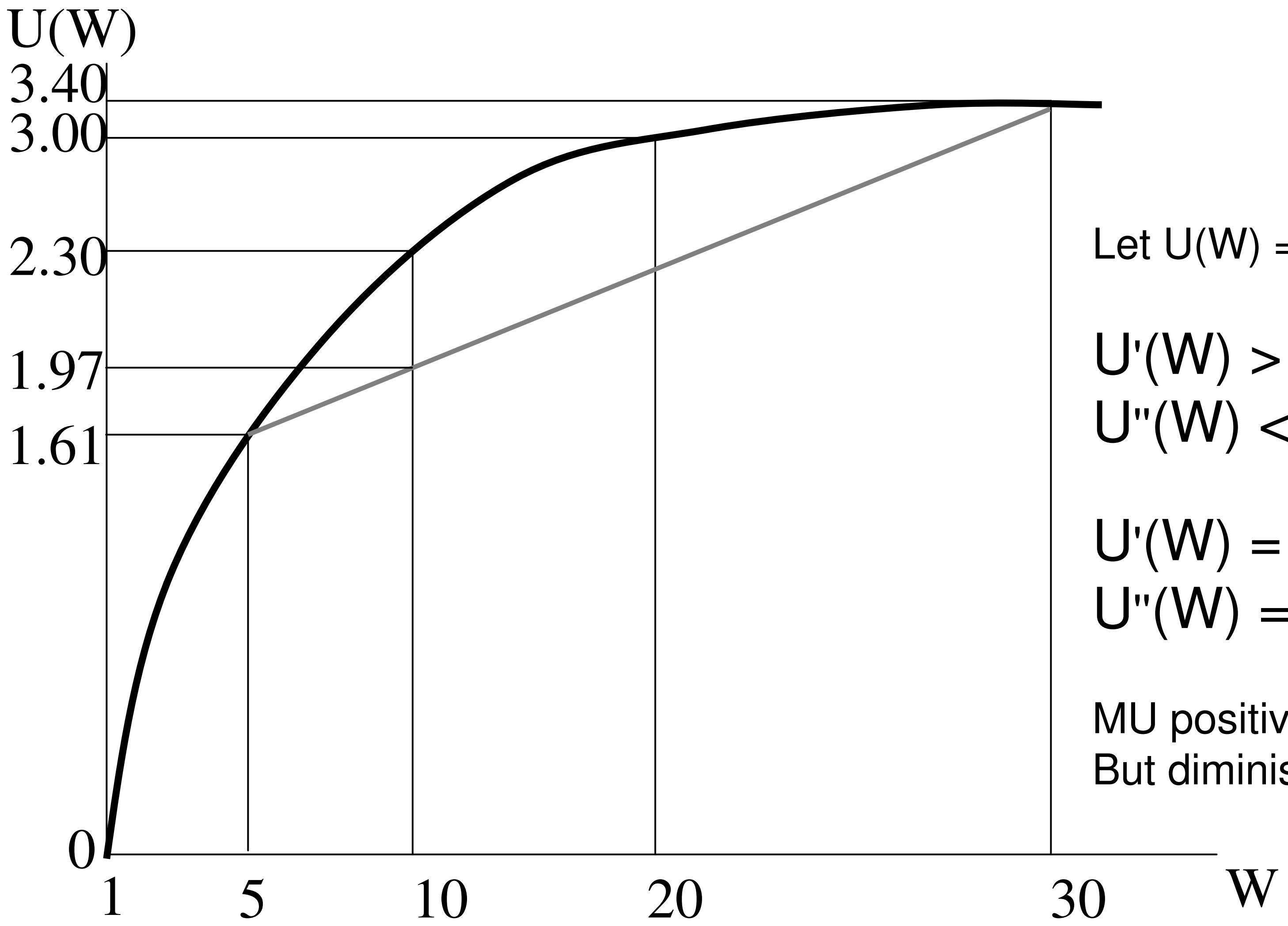
- if you prefer the expected value over the gamble, **risk averse**

# Preferences to Risk




| Risk Preferring | Risk Neutral | Risk Aversion |
| --- | --- | --- |
| $U'(W) > 0$ | $U'(W) > 0$ | $U'(W) > 0$ |
| $U''(W) > 0$ | $U''(W) = 0$ | $U''(W) < 0$ |

# Logarithmic Utility Function




Let $U(W) = \ln(W)$

$U'(W) > 0$
$U''(W) < 0$

$U'(W) = 1/W$
$U''(W) = -1/W^2$

MU positive
But diminishing

# U[E(W)] and E[U(W)]

- U[E(W)] is the utility associated with the known level of expected wealth (although there is uncertainty around what the level of wealth will be, there is no such uncertainty about its expected value)

- E[U(W)] is the expected utility of wealth, that is utility associated with level of wealth that may obtain

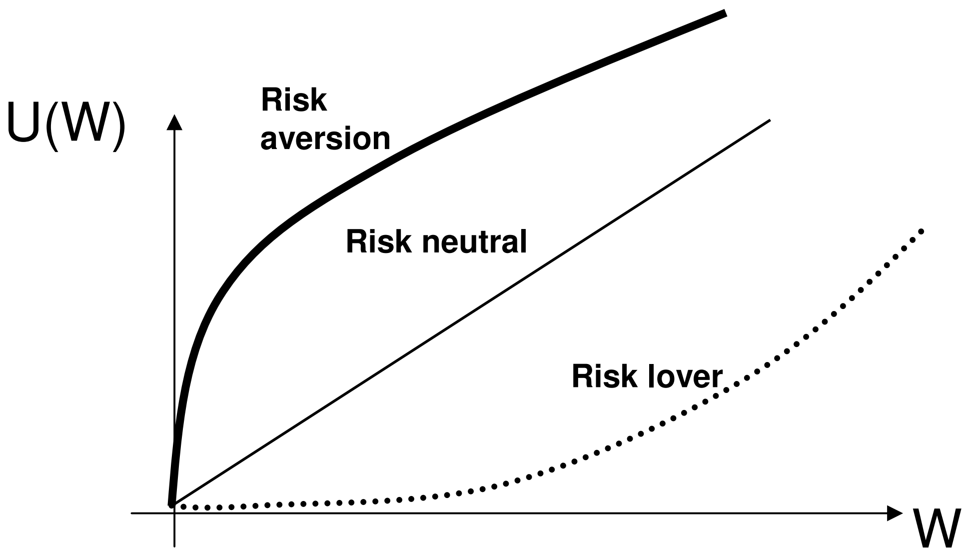
- The relationship between U[E(W)] and E[U(W)] is very important

# Expected Utility, Utility Expected and Risk Aversion

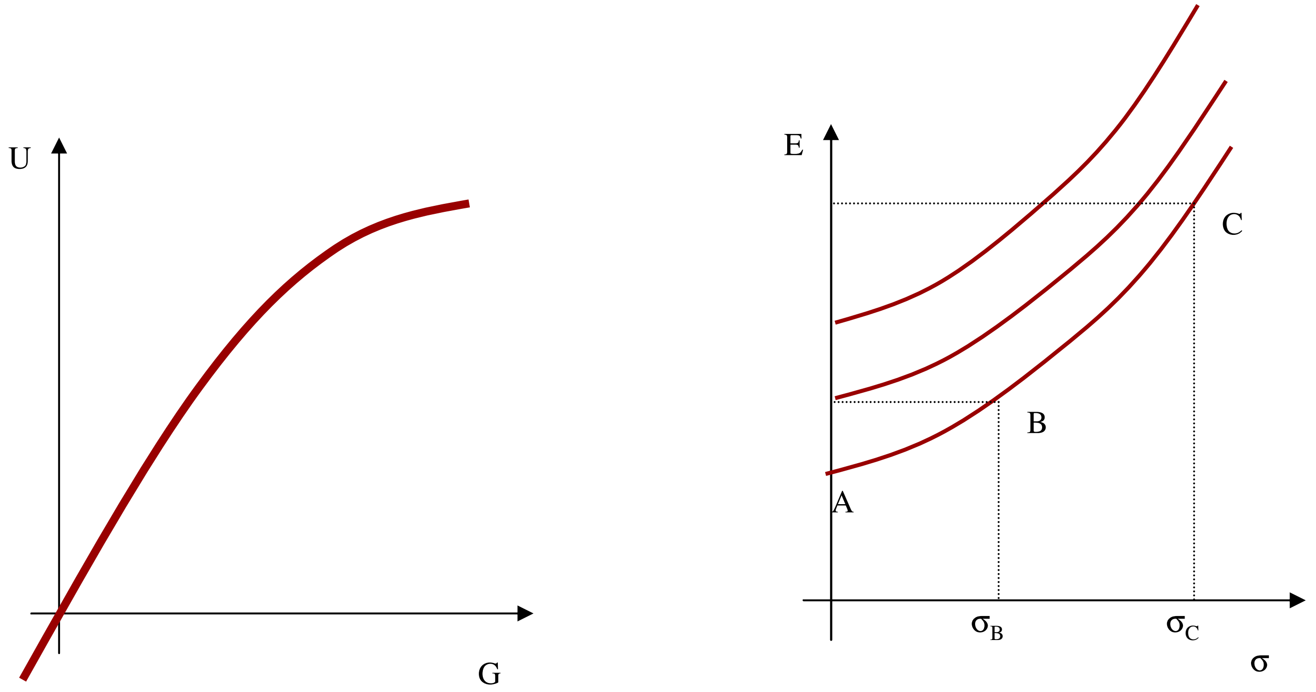
- Risk aversion : U[E(W)] > E[U(W)] : strictly concave utility function

- Risk lover : U[E(W)] < E[U(W)] : strictly convex utility function

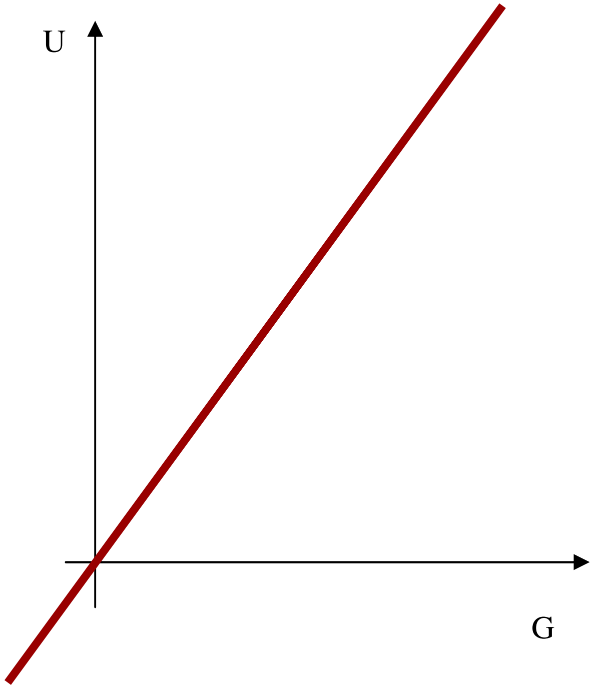
- Risk neutral : U[E(W)] = E[U(W)] : linear utility function

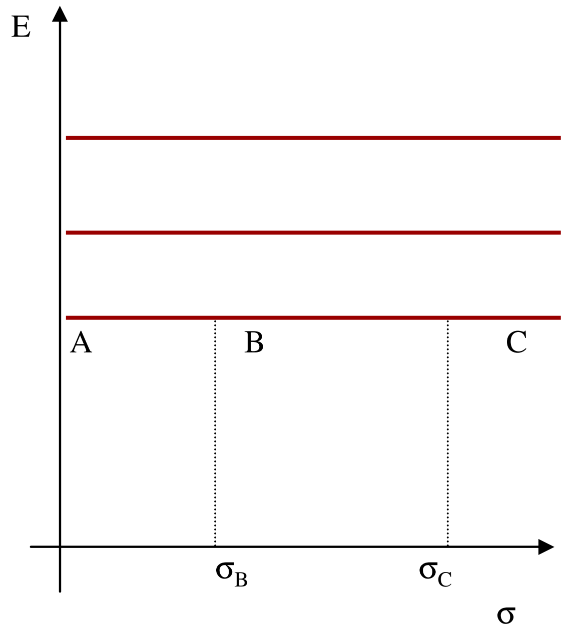


# Indifference curves for a risk-averse investor

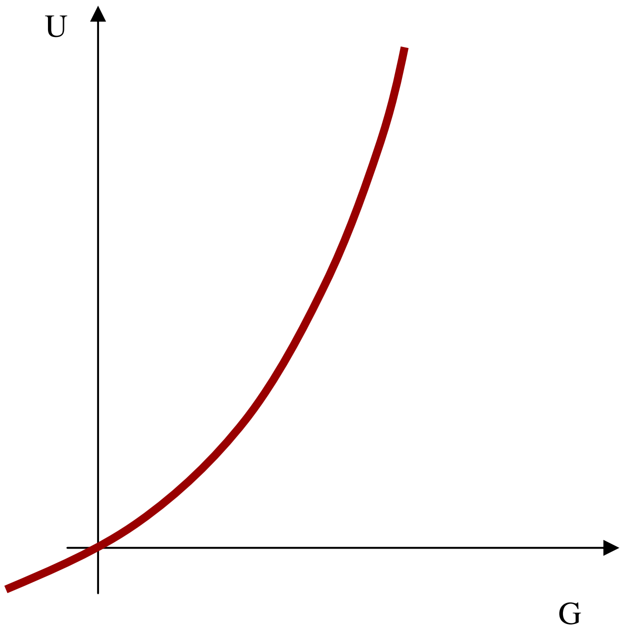


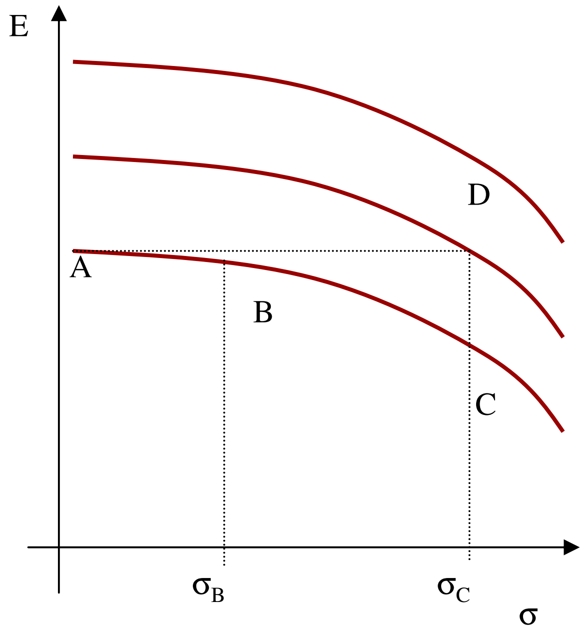
# Indifference curves for a risk-neutral investor



# Indifference curves for a risk-lover investor



# Expected Utility

Assume that the utility function is natural logs: U(W) = ln(W)

Then MU(W) is decreasing

U(W) = ln(W)

U'(W)=1/W => MU>0

U''(W) < 0 => MU diminishing

**Consider the following example:**

80% change of winning €5 and 20% chance of winning €30

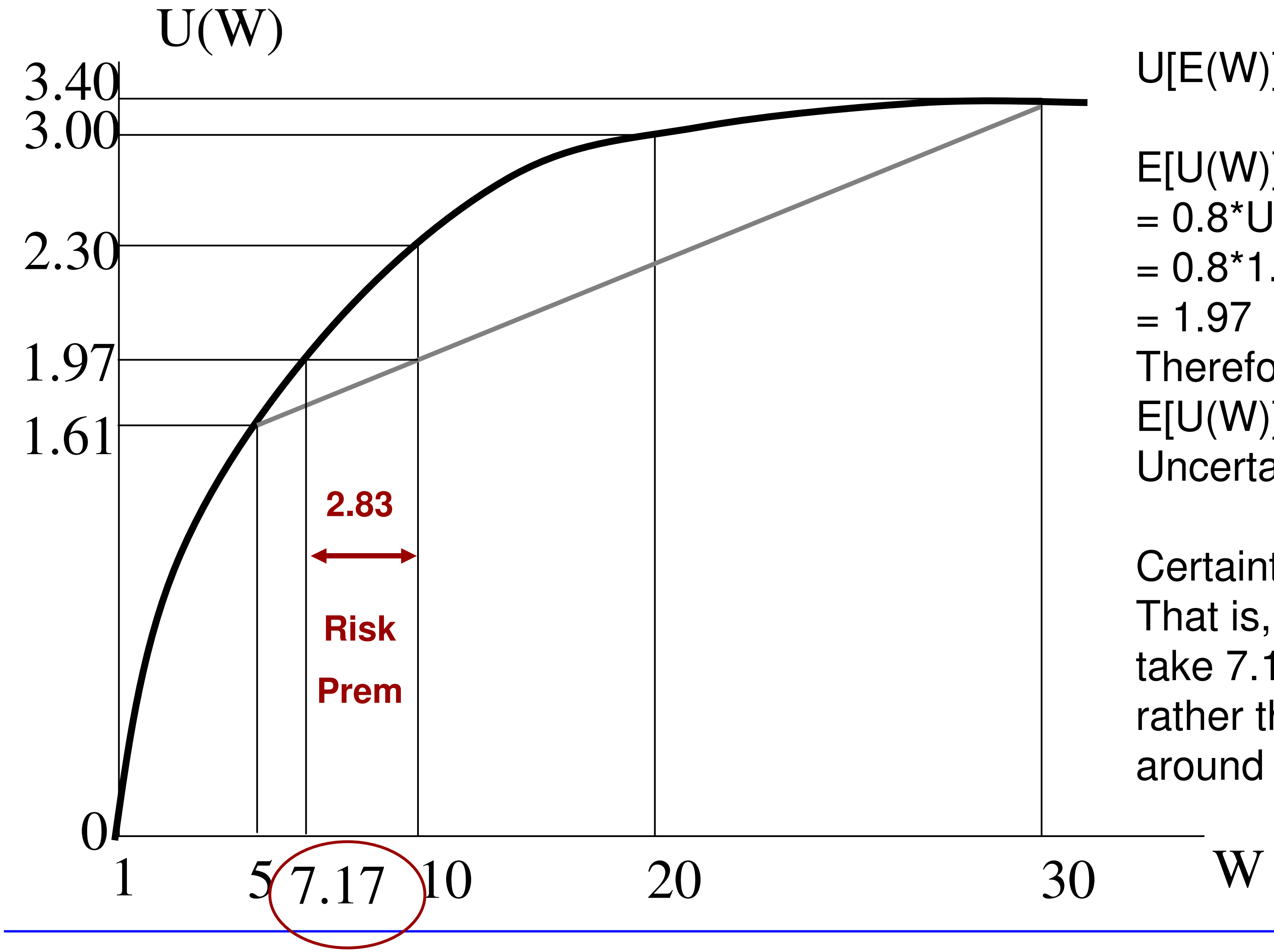
E(W) = (.80)*(5) + (0.2)*(30) = $10

U[E(W)] = U(10) = 2.30

E[U(W)] = (0.8)*[U(5)] + (0.2)*[U(30)]
= (0.8)*(1.61) + (0.2)*(3.40)
= 1.97

Therefore, U[(E(W)] > E[U(W)] -- risk reduces utility

# Certainty Equivalent and Markowitz Premium




U[E(W)] = U(10) = 2.30

E[U(W)]
= 0.8*U(5) + 0.2*U(30)
= 0.8*1.61 + 0.2*3.40
= 1.97
Therefore, U[E(W)] > E[U(W)]
Uncertainty reduces utility

Certainty equivalent: 7.17
That is, this individual will take 7.17 with certainty rather than the uncertainty around the gamble

# The Markowitz Risk Premium

- The Expected wealth is 10
- The E[U(W)] = 1.97
- How much would this individual take with certainty and be indifferent the gamble
- Ln(CE) = 1.97
- Exp(Ln(CE)) = CE = 7.17
- This individual would take 7.17 with certainty rather than the gamble with expected payoff of 10
- The difference, (10 – 7.17 ) = 2.83, can be viewed as a risk premium – an amount that would be paid to avoid risk
- If this individual is offered insurance against the gamble that cost less € 2.83, he will buy it.

# The Risk Premium

| | | | | |
|---|---|---|---|---|
| Risk Premium | = | an individual's expected wealth,given the gamble | - | level of wealth the individual would accept with certainty if the gamble were removed (ie the certainty equivalent) |

- if U[E(W)] > E[U(W)] then risk averse individual (RP > 0)
- if U[E(W)] = E[U(W)] then risk neutral individual (RP = 0)
- if U[E(W)] < E[U(W)] then risk loving individual (RP < 0)

- risk aversion occurs when the utility function is strictly concave
- risk neutrality occurs when the utility function is linear
- risk loving occurs when the utility function is convex

# The Arrow-Pratt Premium

- Risk Averse Investors
- Assume that utility functions are strictly concave and increasing
- Individuals always prefer more to less (MU > 0)
- Marginal utility of wealth decreases as wealth increases

A More Specific Definition of Risk Aversion

W = current wealth
Gamble Z
The gamble has a zero expected value: E(Z) = 0 (*actuarially neutral*)

what risk premium $\pi(W,Z)$ must be added to the gamble to make the individual indifferent between the gamble and the expected value of the gamble?

# The Arrow-Pratt Premium

The risk premium $\pi$ can be defined as the value that satisfies the following equation:

$$E[U(W+Z)] = U[W + E(Z) - \pi(W, Z)] \quad (1)$$

LHS:
expected utility of
the current level
of wealth, given the
gamble

RHS:
utility of the current level of wealth
plus
the expected value of
the gamble
less
the risk premium

We want to use a Taylor series expansion to (1) to derive an expression for the risk premium $\pi(W,Z)$

# Absolute Risk AVersion

- Arrow-Pratt Measure of a Local Risk Premium (derived from (1) above):

$$\pi = \frac{1}{2}\,\sigma_Z^2 \left( -\frac{U''(W)}{U'(W)} \right)$$

- Define ARA as a measure of Absolute Risk Aversion

$$ARA = -\frac{U''(W)}{U'(W)}$$

- This is defined as a measure of absolute risk aversion because it measures risk aversion for a given level of wealth
- ARA > 0 for all risk averse investors (U'>0, U''<0)
- How does ARA change with an individual's level of wealth?
- ie. a €1000 gamble may be trivial to a rich man, but non-trivial to a poor man

=> ARA will probably decrease as our wealth increases

# Relative Risk Aversion

- Constant RRA => An individual will have constant risk aversion to a "proportional loss" of wealth, even though the absolute loss increases as wealth does
- Define RRA as a measure of Relative Risk Aversion

$$RRA = -W * \frac{U''(W)}{U'(W)}$$

# CARA, IARA, DARA

- Consider some utility function and its associated absolute risk aversion as a function of wealth, $A(W)$.
- We say that absolute risk aversion is decreasing (DARA) if $A'(W)<0$; it is constant (CARA) if $A'(W)=0$; it is increasing (IARA) if $A'(W)>0$.
- Consider then two people, both with the same utility function, but one is very poor, the other very rich.
- Suppose also that both agents are subject to the same risk of losing, say, \$10.
- Who of the two is more willing to insure against this risk?

# CARA, IARA, DARA

- It seems reasonable to assume that the rich person is not willing to insure, because he will not notice the difference of $10.
- The very poor person, on the other hand, may be very eager to insure, because $10 amounts to a substantial part of his wealth.
- If this is the case, then utility is DARA.
- IARA would imply that the rich person buys insurance against this risk, but the poor does not, which seems not plausible.

## Some Standard Utility Functions

- Quadratic Utility Function:

$$U(W) = W - aW^2$$

$$U'(W) = 1 - 2aW > 0$$

$$U''(W) = -2a$$

Only for : $W < \dfrac{1}{2a}$

ARA = $\dfrac{2a}{1-2aW}$ (dARA/dW>0) ...not intuitive

RRA = $\dfrac{2aW}{1-2aW}$ (dRRA/dW>0)

# Some Standard Utility Functions

- <u>Exponential Utility Function</u>:

$$U(W) = -e^{-aW} \qquad a \geq 0$$

$$U'(W) = a e^{-aW} > 0$$

$$U''(W) = -a^2 e^{-aW} < 0$$

ARA = $a$ (dARA/dW=0)

RRA = $aW$ (dRRA/dW>0)

# Some Standard Utility Functions

- Logarithmic Utility Function (Bernoulli):

$$U(W) = \ln W$$

$$U'(W) = W^{-1}$$

$$U''(W) = -W^{-2}$$

ARA = $W^{-1}$ (dARA/dW<0)

RRA = $1$ (dRRA/dW=0)

# Some Standard Utility Functions

- Power Utility Function:

$$U(W) = W^b \qquad 0 < b < 1$$

$$U'(W) = bW^{b-1}$$

$$U''(W) = (b-1)bW^{b-2}$$

ARA = $-\dfrac{(b-1)bW^{b-2}}{bW^{b-1}} = \dfrac{1-b}{W}$ (dARA/dW<0)

RRA = $1-b$ (dRRA/dW=0)

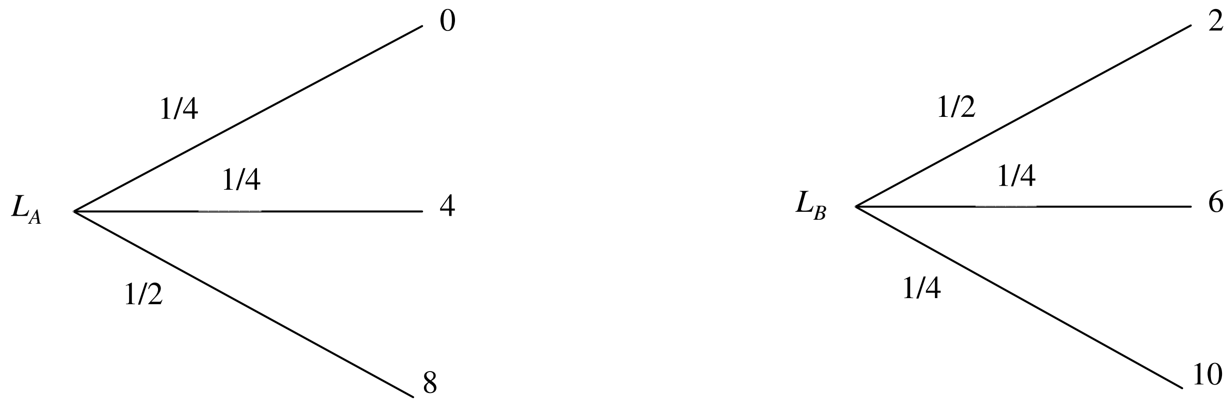
# Animal Experiments

- Kagel et al. (1995) have performed interesting experiments with animals.
- Payoff is food.
- Very good control over "wealth" (nutritional level) of animals.
- They find evidence for risk aversion and DARA.
- In fact, they find some weak evidence for a utility function of the form $(x\text{-}s)^{1-\gamma}/(1-\gamma)$, where $x$ is consumption and $s>0$ can be interpreted as subsistence level (see chapter 6.3 of their book).
- This utility function exhibits DRRA as well.

# Empirical Evidence

- Friend and Blume (1975): study U.S. household survey data in an attempt to recover the underlying preferences. Evidence for DARA and almost CRRA, with $R \approx 2$.
- Tenorio and Battalio (2003): TV game show in which large amounts of money are at stake. Estimate rel. risk aversion between 0.6 and 1.5.
- Abdulkadri and Langenmeier (2000): farm household consumption data. They find significantly greater risk aversion.
- Van Praag and Booji (2003): large survey done by a dutch newspaper. They find that rel risk aversion is close to log-normally distributed, with a mean of 3.78.

# Prudence, Temperance and Edginess




- Same expected gain (5) and same variance (11)

- Which prospect do you prefer ?

# Prudence and Downside Risk Aversion

$$\left.\begin{array}{l} U''' > 0 \Rightarrow L_B \succ L_A \\ U''' < 0 \Rightarrow L_B \prec L_A \\ U''' = 0 \Rightarrow L_B \approx L_A \end{array}\right\}$$

Downside Risk Aversion
(or equivalently prudence)

Prudence coefficient $= -\frac{U'''}{U''}$ DR aversion, skewness preference

Temperance coefficient $= -\frac{U''''}{U'''}$ Behavior towards a risk in presence of a second unavoidable risk

Edginess coefficient $= -\frac{U'''''}{U''''}$ Reactivity to multiple risks

# An example: the Log-Utility Function

$$U(W) = \ln W$$

$$U'(W) = \frac{1}{W} > 0$$

$$U''(W) = -\frac{1}{W^2} < 0$$

$$U'''(W) = \frac{2}{W^3}$$

$$U''''(W) = \frac{-6}{W^4}$$

$$U'''''(W) = \frac{24}{W^5}$$

$$\mathrm{ARA} = -\frac{U''(W)}{U'(W)} = W^{-1}$$

$$\mathrm{P} = -\frac{U'''(W)}{U''(W)} = 2W^{-1}$$

$$\mathrm{T} = -\frac{U''''(W)}{U'''(W)} = 3W^{-1}$$

$$\mathrm{H} = -\frac{U'''''(W)}{U''''(W)} = 4W^{-1}$$

# An example about premiums

- U=ln(W)    W = $20,000
- G(10,-10: 50) 50% will win 10, 50% will lose 10
- What is the risk premium associated with this gamble?
- Calculate this premium using both the Markowitz and Arrow-Pratt Approaches

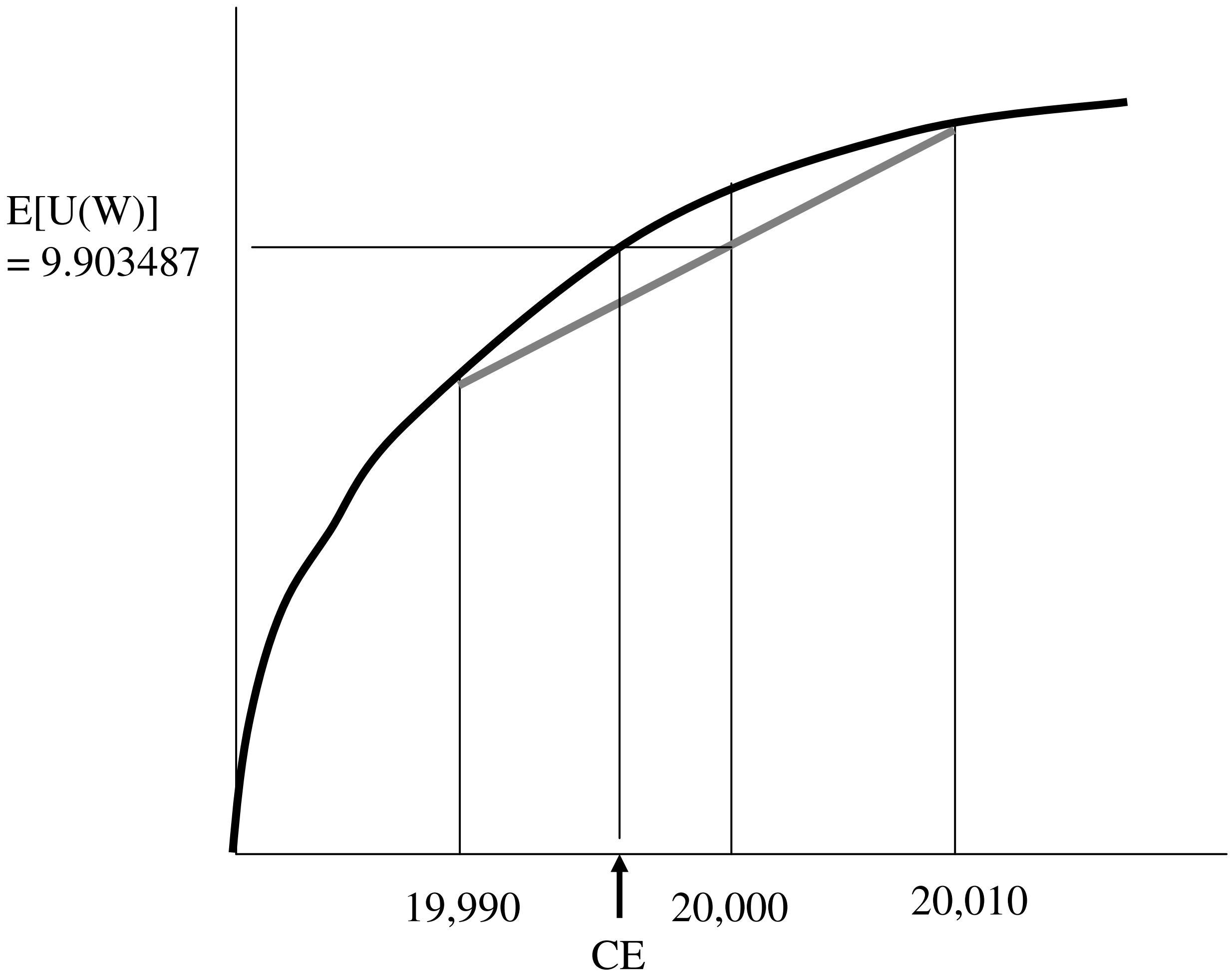
## Arrow-Pratt Measure

- $\pi = -(1/2)\ \sigma^2_z\ U''(W)/U'(W)$
- $\sigma^2_z = 0.5*(20{,}010 - 20{,}000)^2 + 0.5*(19{,}990 - 20{,}000)^2 = 100$
- $U'(W) = (1/W) \qquad U''(W) = -1/W^2$
- $U''(W)/U'(W) = -1/W = -1/(20{,}000)$
- $\pi = -(1/2)\ \sigma^2_z\ U''(W)/U'(W) = -(1/2)(100)(-1/20{,}000) =$ \$0.0025

# Markowitz Measure

- $E(U(W)) = \Sigma\, p_i U(W_i)$
- $E(U(W)) = (0.5)U(20{,}010) + 0.5*U(19{,}990)$
- $E(U(W)) = (0.5)\ln(20{,}010) + 0.5*\ln(19{,}990)$
- $E(U(W)) = 9.903487428$
- $\ln(CE) = 9.903487428 \rightarrow CE = 19{,}999.9975$
- The risk premium RP = $0.0025
- **Therefore, the AP and Markowitz premia are the same**

# Markowitz Measure



# Empirical Differences of two Approaches

- Markowitz premium is an exact measures whereas the AP measure is approximate
- AP assumes symmetry payoffs across good or bad states, as well as relatively small payoff changes.
- It is not always easy or even possible to invert a utility function, in which case it is easier to calculate the AP measure
- The accuracy of the AP measures decreases in the size of the gamble and its asymmetry

# 2.4 Stochastic Dominance

## A General Efficiency Criterion

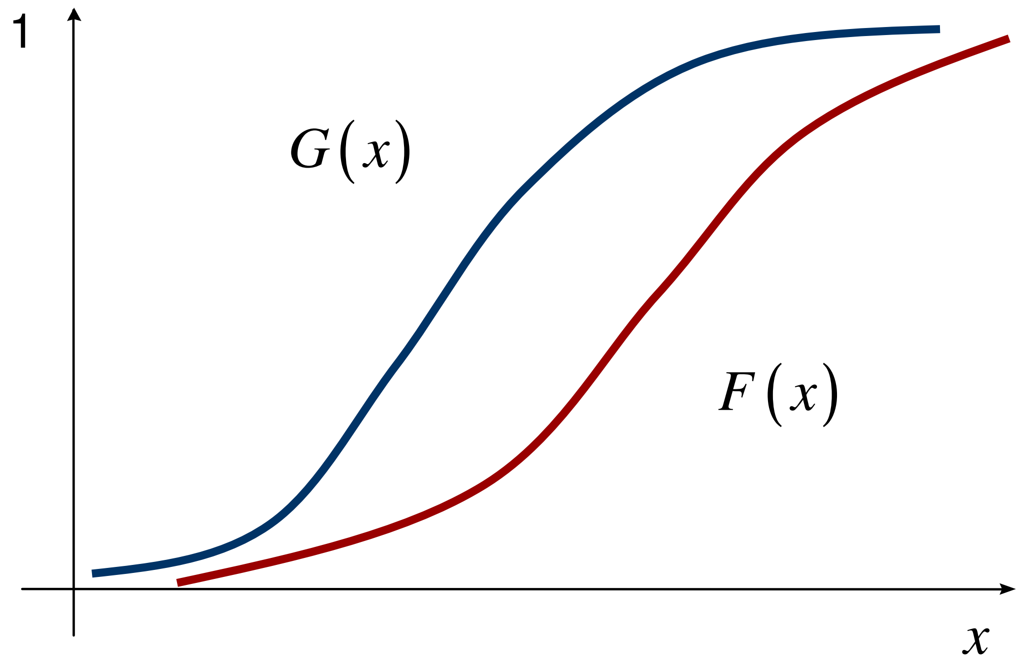
- The "more" risk averse someone is the less likely they are to engage in a gamble
- The notion of "more" risk averse is hard to quantify however, and requires precise utility functions, which in practice are hard to calculate.
- The idea of stochastic dominance eliminates the need to calculate utility functions
- The most general efficiency criteria relies only on the assumption that utility is nondecreasing in income

# First-order Stochastic Dominance

- Given two cumulative distribution functions F and G, an option F will be preferred to the second option G by FSD if $F(x) \leq G(x)$ for all return x with at least one strict inequality.
- An asset is said to be stochastically dominant over another if an individual receives greater wealth from it in every (ordered) state of nature
- Only one assumption: the utility function is increasing in wealth

# First-order Stochastic Dominance

• Intuitively, this rule states that one alternative F will dominate G if F lies under G at all points



# A Numerical Example of First-order Stochastic Dominance

Assume two random variables X & Y with probability distributions as follows:

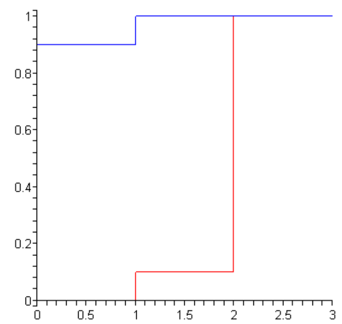
| | Outcomes | | |
|---|---|---|---|
| | Low profit = 1 | Average Profit = 2 | High Profit = 3 |
| X | 0 | 0.1 | 0.9 |
| Y | 0.9 | 0.1 | 0 |

In this case z takes on three values: 1, 2 and 3. For X to stochastically dominate Y recall $F(z) \leq G(z)$ For all $z$

| *Z* | *F(z)* | | *G(z)* |
|---|---|---|---|
| 1 | 0 | < | .9 |
| 2 | .1 | < | 1 |
| 3 | 1 | = | 1 |

Since *F(z)* is always less than or equal to *G(z)*, X dominates Y

*F(z)* is in read
*G(z)* is in blue



# Second-order Stochastic Dominance

• Stochastic variable X with cumulative distribution function (CDF) *F* dominates stochastic variable Y with CDF *G* if:

$$\int_{-\infty}^{x} F(z)\,dz \le \int_{-\infty}^{x} G(z)\,dz \quad \text{or} \quad \int_{-\infty}^{x} \left[G(z) - F(z)\right] dz \ge 0$$

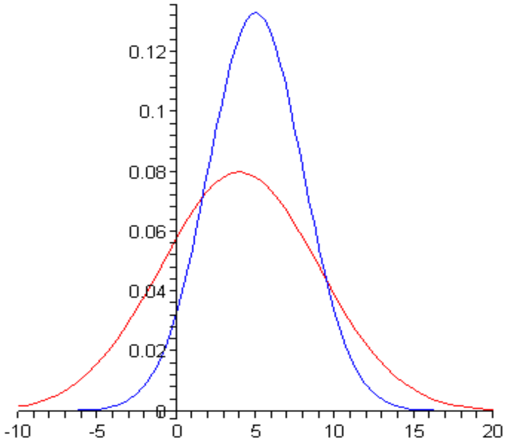
• This inequality implies that the distribution of Y has more weight at lower values of wealth than does X and this more than offsets the possible higher weights of the distribution of Y at high values of wealth

• **If the utility function is increasing in wealth and it is concave, then the individual will prefer F to G.**

# Graphical Representation

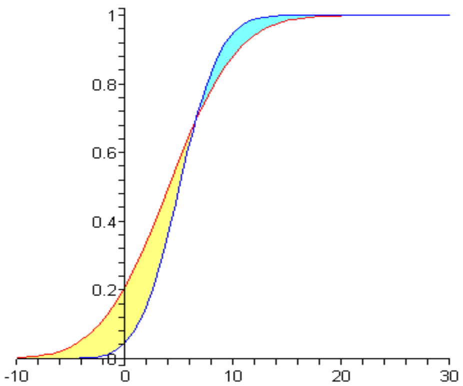
The blue curve is the density function of X

The red curve is the density function of Y




The blue curve is the cumulative distribution of X

The red curve is the cumulative distribution of Y



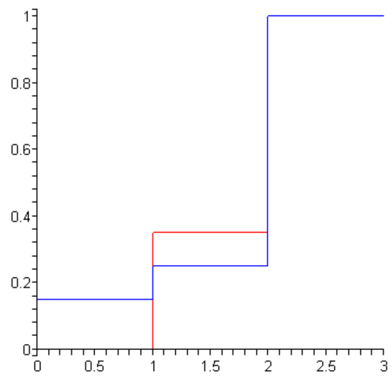

X dominates Y because the yellow area in the cumulative distribution graph is larger than the blue area

# A Numerical Example of Second-order Stochastic Dominance

| | Outcomes | | |
|---|---|---|---|
| | Low profit = 1 | Average Profit = 2 | High Profit = 3 |
| X | 0 | 0.35 | 0.65 |
| Y | 0.15 | 0.1 | 0.75 |

Again z takes on three values: 1, 2 and 3. For X to stochastically dominate Y recall: $S(z)=\int(F(q)-G(q))dq \leq 0$ For all $z$

| Z | F(z) | G(z) | F(z) - G(z) | S(z) |
|---|---|---|---|---|
| 1 | 0 | .15 | -.15 | -.15 |
| 2 | .35 | .25 | .1 | -.05 |
| 3 | 1 | 1 | 0 | -.05 |




Since ∑[F(z)-G(z)]<0 for all outcomes, X stochastically dominates Y

*F(z)* is in read
*G(z)* is in blue

# Mean, Variance and SD criteria

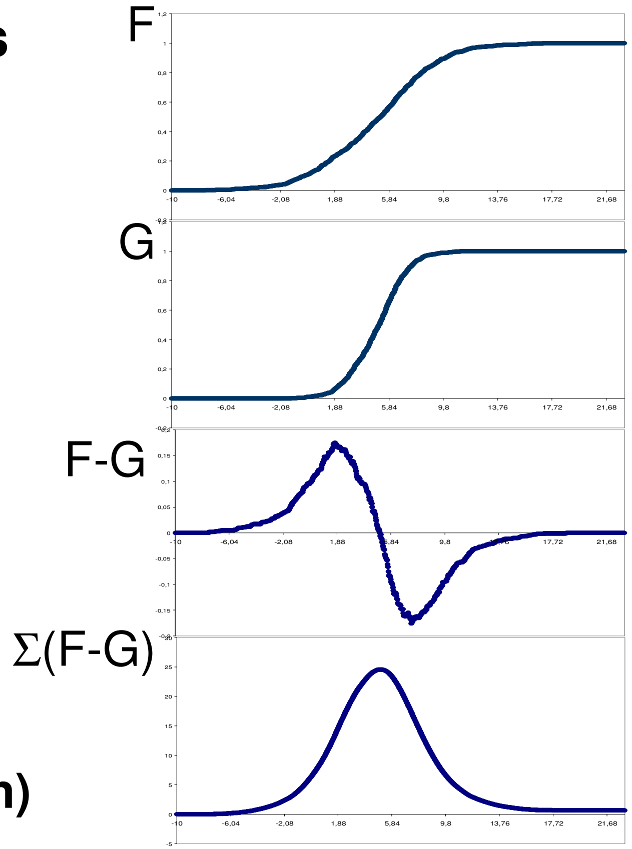
- **Consider two normally distributed variables**

- E(A) = 5 σ(A)=4

- E(B) = 5 σ(B)=2

- F is the CDF of A

- G is the CDF of B




- **FOD : no conclusion ; CDF cross**

- **SOD : B dominates A (lower variance same mean)**

# Mean, Variance and SD criteria

- **Consider two normally distributed variables**

- E(A) = 5  σ(A)=4

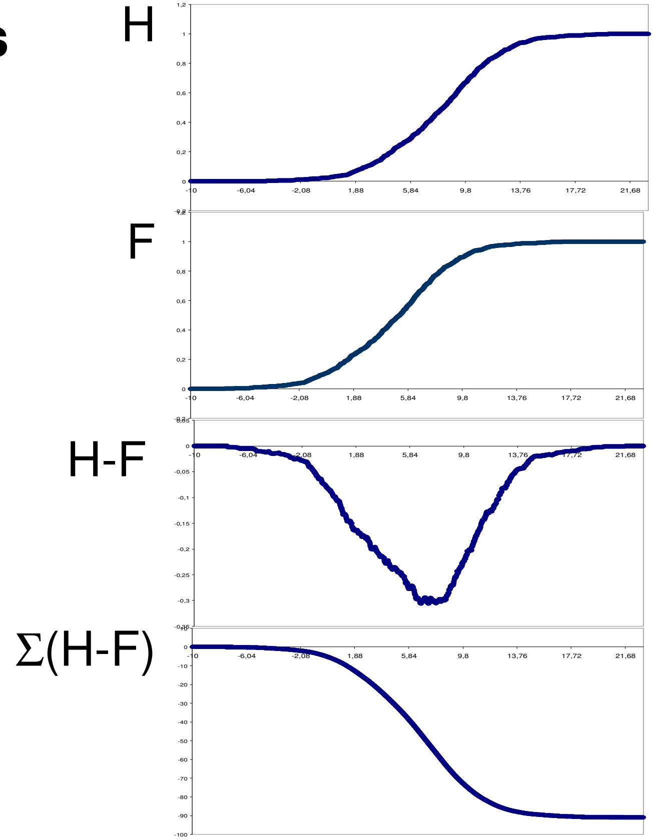
- E(C) = 8  σ(C)=4

- F is the CDF of A

- H is the CDF of C

- **FOD : C dominates A (H<F)**

- **SOD : C dominates A (ΣH < ΣF)**



# Mean, Variance and SD criteria

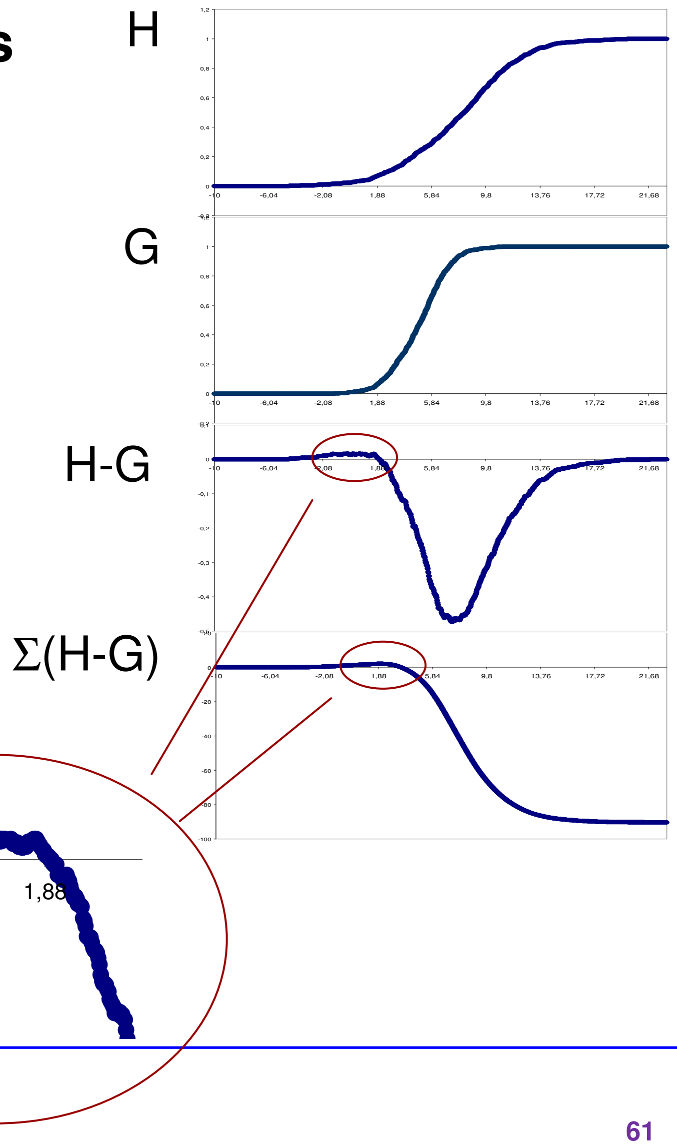
- **Consider two normally distributed variables**

- E(B) = 5    σ(B)=2

- E(C) = 8    σ(C)=4

- G is the CDF of B

- H is the CDF of C




- **FOD : no conclusion**

- **SOD : no conclusion**

# 2.5 Non-Expected Utility

# Allais Paradox

- Consider a set C with three possible outcomes

$$C = \{\text{Win } \$4000, \text{Win } \$3000, \text{Win } \$0\}$$

- Consider a scenario A which is a choice between the following two lotteries

$$L_A = (.8, 0, .2) \qquad L'_A = (0, 1, 0)$$

and a second scenario B which is a choice between following two lotteries

$$L_B = (.2, 0, .8) \qquad L'_B = (0, .25, .75)$$

# Allais Paradox

- Most individuals will select lottery $L'_A$ in scenario A and lottery $L_B$ in scenario B
  - Lottery $L'_A$ provides opportunity to win $3000 with certainty
  - Almost always win nothing, but lottery $L_B$ provides highest chance of winning the greatest dollar amount

- Choice of $L'_A$ and $L_B$ violates EUT

Let $u(0) = 0$, then by EUT $L'_A$ is preferred to $L_A$ if and only if $u(3000) > 0.8u(4000)$

By Proposition 2, this is equivalent to $.25u(3000) > .2u(4000)$ which implies that $L'_B$ is preferred to $L_B$ which is a contradiction

# Framing Effects – Kahneman and Tversky

- Respondents given statistical information about two treatments of lung cancer – surgery and radiation therapy
  - Surgery:
    - Of 100 people having surgery, 90 live through the post-operative period, 68 are alive at the end of the first year and 34 are alive at the end of five years
  - Radiation Therapy:
    - Of 100 people having radiation therapy all live through the treatment, 77 are alive at the end of one year and 22 are alive at the end of five years
- Information would translate into survival probabilities
  - Surgery:
    - Of 100 people having surgery, 10 die during the surgery or the post-operative period. 32 die by the end of the first year and 66 die by the end of five years.
  - Radiation Therapy:
    - Of 100 people having radiation therapy none died during the treatment. 23 die by the end of one year and 78 die by the end of five years

# Framing Effects – Kahneman and Tversky

- Respondents in the K-T experiment choose radiation therapy more often when described in the “mortality frame” than in “survival frame”
  - In “survival frame” 18% chose radiation therapy (N = 247)
  - In “mortality frame” 44% chose the radiation therapy (N = 336)
- Risk of immediate death evidently looms larger when described as such
- Results suggest how lottery is described or “framed” can affect choices
  - Framing effect was not smaller for experienced physicians or business school students than a group of clinic patients

# Risk aversion?

A) An investment in bonds that pays $50 for sure.

*risk-seeking (low p gain)*

*B) An investment in an oil wildcatting project that offers: a 5% chance to receive $1,000
a 95% chance of receiving nothing

*C) Pay $1,000 to insure a $20,000 factory robot that has a 5% chance of being destroyed.

*risk-aversion (low p loss)*

D) Risk a 5% chance of your $20,000 factory robot being destroyed.

E) A 95% chance of a gold claim in which you have invested pays $100,000
(but a 5% chance that it is worthless).

*risk-aversion (high p gain)*

*F) Accept a $95,000 offer to buy out your claim.

G) Pay the IRS $9,500 in back taxes they claim you owe them.

*risk-seeking (high p loss)*

*H) Appeal the decision and face a: 5% chance of paying nothing and
95% chance of paying $10,000.

# Comparative Ignorance: Relative Knowledge

1a) The two leading candidates in this year's election for president of the United States are Bill Clinton and Bob Dole.

*Who do you think will win? (Circle one)*

Clinton Dole

1b) The two leading candidates in this year's election for president of the Dominican Republic are Leonel Fernández and José Francisco Peña Gómez.

*Who do you think will win? (Circle one)*

Fernández Gómez

## Comparative Ignorance: Relative Knowledge

2) The two leading candidates for president of Russia are Boris Yeltsin and Gennady Zyuganov.

*Who do you think will win? (Circle one)*

Yeltsin Zyuganov

3) Which would you rather receive? (Circle one)

$50 for sure

$150 if I am right about the Russian election

| | | |
|---|---|---|
| ***A:*** | ***U.S. ... Russia*** | ***28% chose $150 if right*** |
| ***B:*** | ***D.R. ... Russia*** | ***62% chose $150 if right*** |

# Descriptive Perspective: Prospect Theory

According to EU people make choices that maximize

$$EU = \Sigma p_i U(x_i)$$

*Expected Utility* — *probability* — *utility of receiving $x*

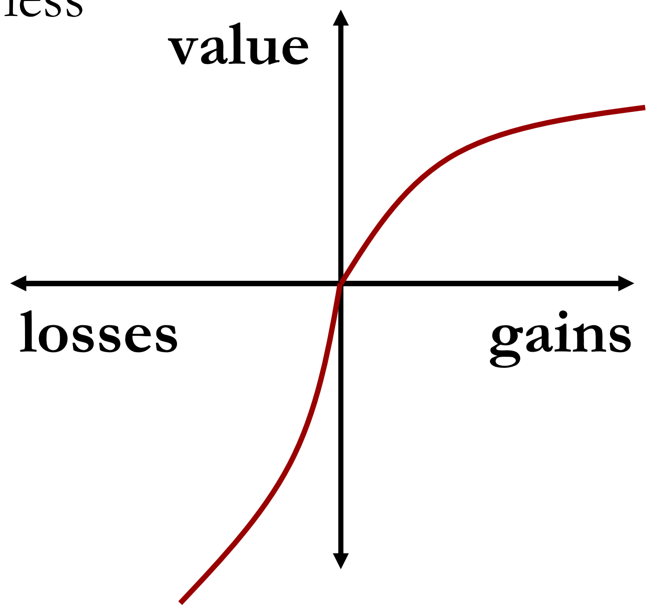
According to PT people make choices that maximize

$$V = \Sigma w(p_i) W(x_i)$$

*Value of the prospect* — *decision weight of probability* — *value of receiving $x*

## Value in Prospect Theory (Kahneman & Tversky, 1979

- **Reference dependence:** people are sensitive to losses and gains relative to their reference state (e.g., the status quo). The reference state can be manipulated through framing.
- **Diminishing sensitivity:** people are less and less sensitive to each additional dollar gained or lost (i.e., the value function is concave for gains and convex for losses).
- **Loss aversion:** losses loom larger than gains (i.e., the value function is steeper for losses than for gains).



## Consequences of the Value Function

### 1) Loss Aversion

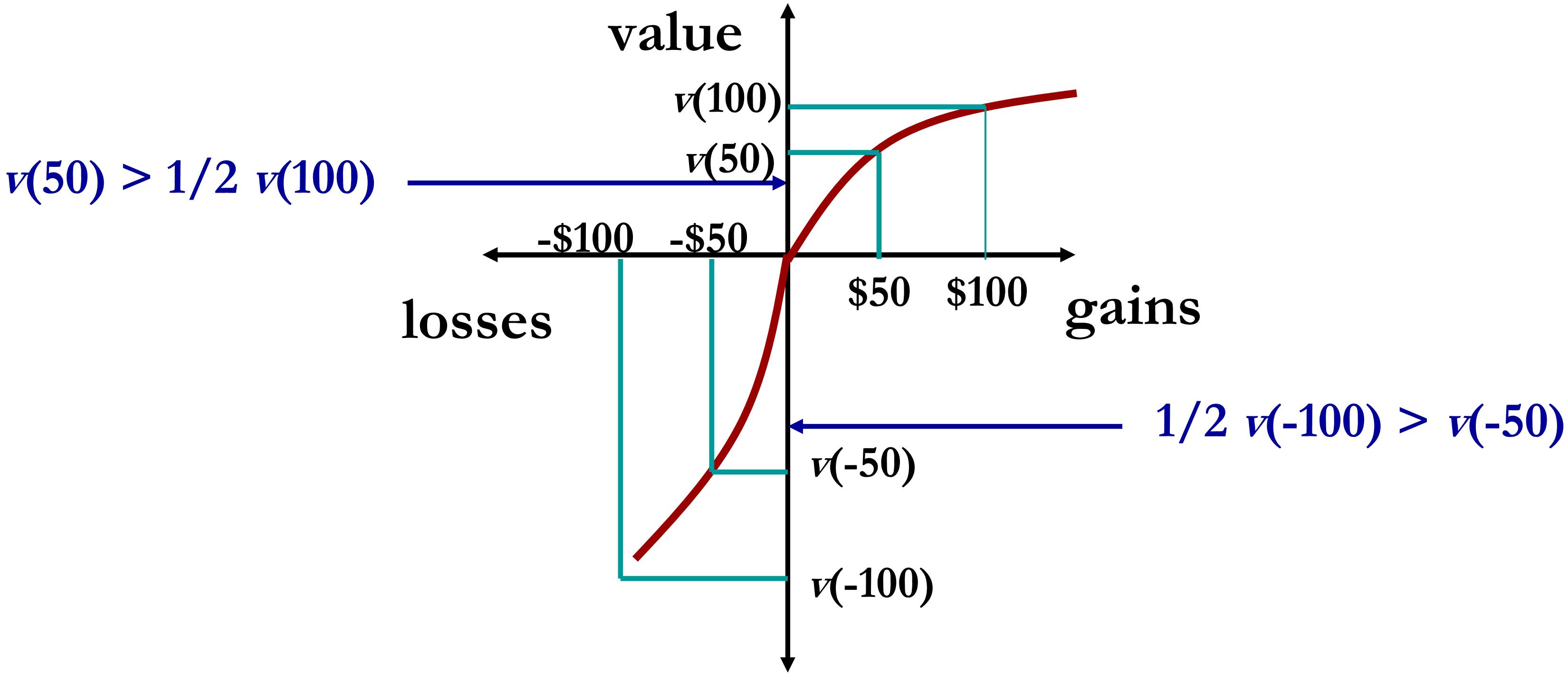
- Losses have greater psychological impact than foregone gains.
- The perception of an outcome as gain or loss is influenced by the way in which the decision is framed.

# Consequences of the Value Function

## 2) The Reflection Effect

- People tend to be risk-averse for gains but risk-seeking for losses.
- The perception of gains vs. losses is influenced by the way in which the problem is framed.

# Gain-Loss Framing Effect



# Real World Portfolios

- Behavioral portfolios contain both very safe (e.g. cash) and highly risky assets (e.g. options).
- Cash, bonds and equities are the most common elements of investors' portfolios.
- **Portfolio puzzle**:
  Investment advisors recommend increasing the ratio of equities to bonds in order to increase the aggressiveness of a portfolio.
  → Violation the two-fund separation theorem of CAPM.
  (keep ratio of equities to bonds constant but change proportion of risk-free asset)

Thank you for your attention…

See you next week