Portfolio Choice

2009/2010

Session 4

Christophe BOUCHER

christophe.boucher@univ-paris1.fr

4.1 Returns Behavior and the Bell-Curve hypothesis

4.2 Volatility: Traditional Measure of Risk

4.3 Alternative Risk Measures

4.4 Lower Partial Moments

4.5 VaR and the Expected Shortfall

4.6 Geometric Mean and Safety First Criteria

4.1 Returns Behavior and the Bell-Curve hypothesis

Returns Behavior and the Bell-Curve hypothesis

- The normal distribution, also called the Gaussian distribution, is an important family of continuous probability distributions.

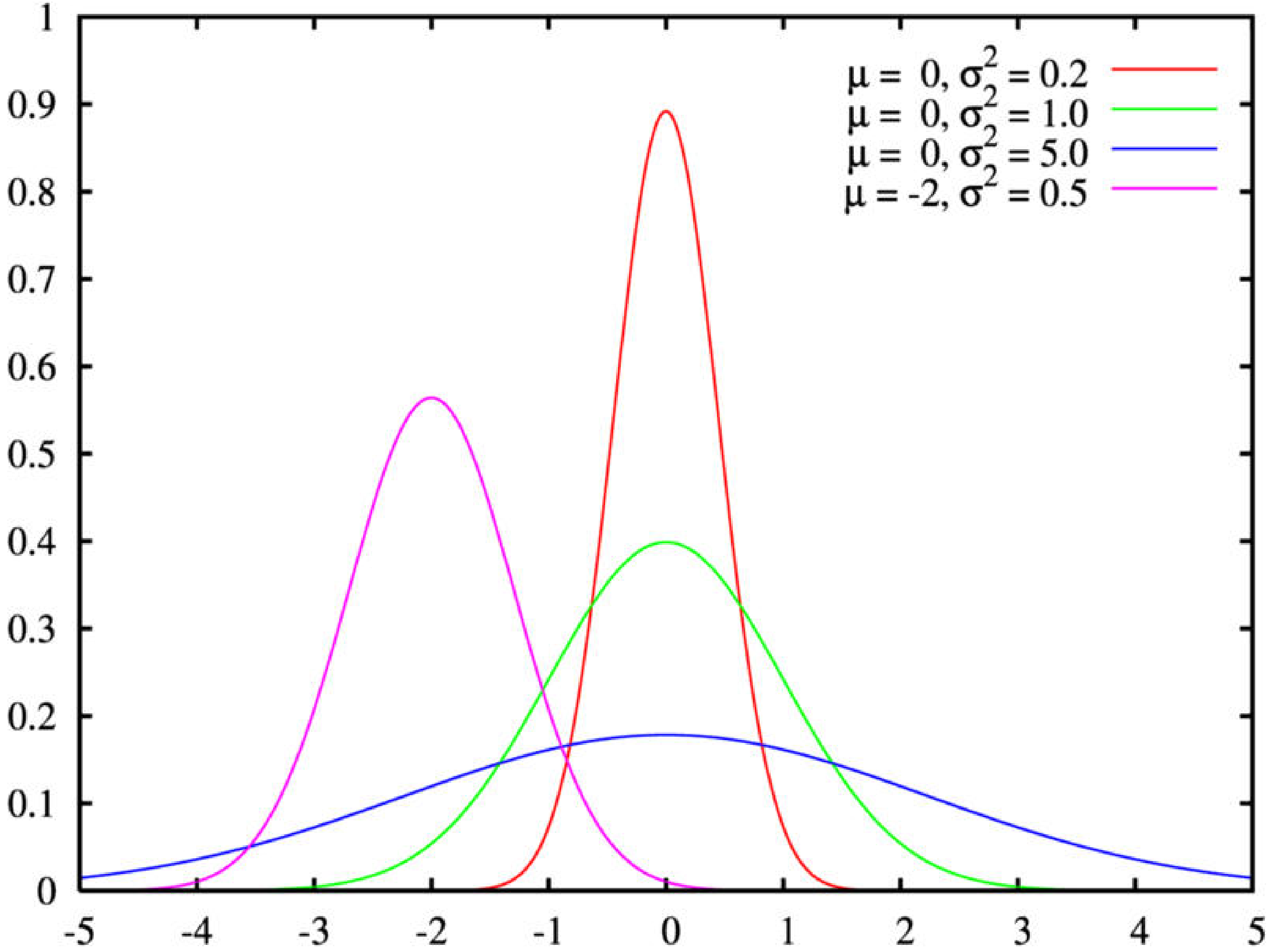
- Defined by two parameters, location and dispersion:
  - mean ("average", μ)
  - variance (standard deviation squared, σ^2)

- The standard normal distribution is the normal distribution with a mean of zero and a variance of one

- The "bell-curve" (shape of the probability density) is used as approximation of many psychological, physical, social or biological phenomena (central limit theorem)

- The probability density function: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Bell-Curves



$\mu = 0, \sigma^2 = 0.2$
$\mu = 0, \sigma^2 = 1.0$
$\mu = 0, \sigma^2 = 5.0$
$\mu = -2, \sigma^2 = 0.5$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-5
-4
-3
-2
-1
0
1
2
3
4
5

Characteristics and properties of the normal density function

- Mean = Median = Mode $\Rightarrow$ Maximum of the density function

- $-\infty < X < \infty$

- The area under the curve is equal to 1

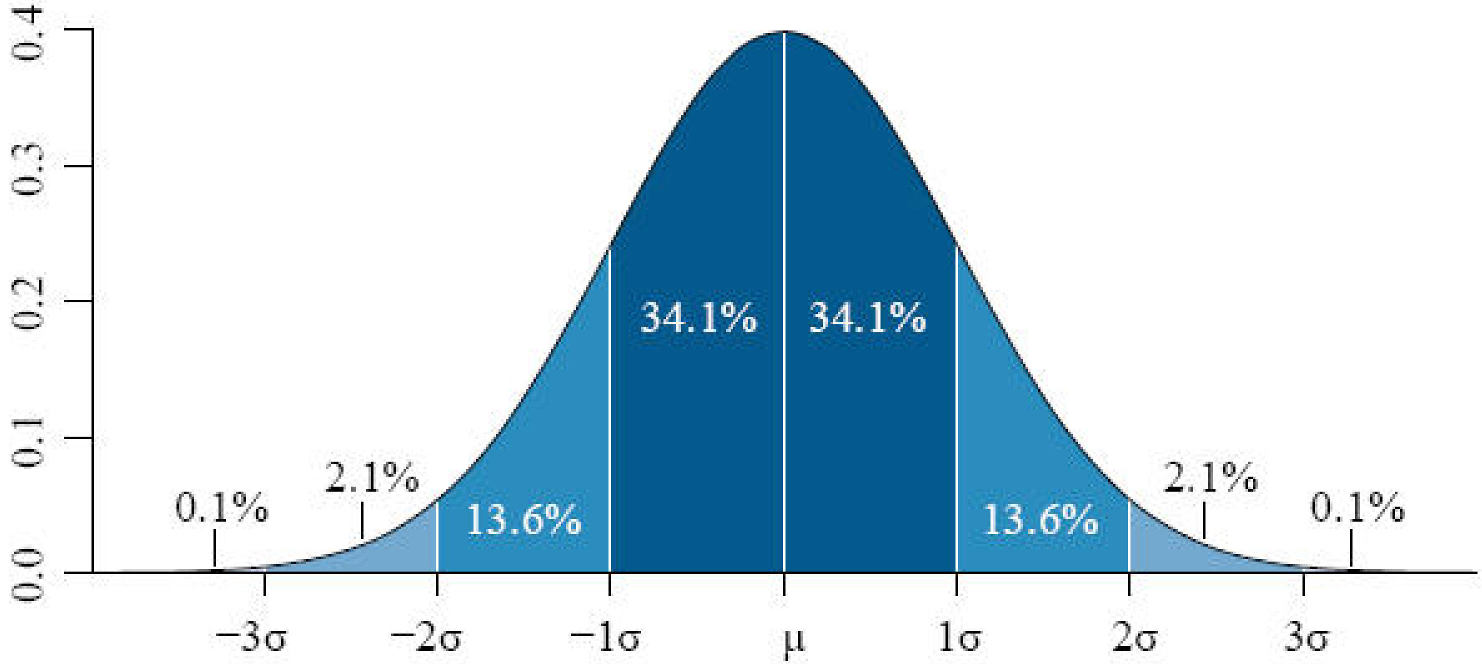
- Symmetry about its mean μ

- The inflection points of the curve occur one standard deviation away from the mean, i.e. at $\mu - \sigma$ and $\mu + \sigma$.

- 68-95-99.7 rule

- $X \sim N(\mu, \sigma) \quad \Rightarrow \quad aX + b \sim N(a\mu + b, a\sigma)$

Dispersion and the Bell Curve (confidence intervals)



$\mu - 0.5\,\sigma$ < 38,1% obs. < $\mu + 0.5\,\sigma$.
$\mu - 1\,\sigma$ < *68,3%* obs. < $\mu + 1\,\sigma$.
$\mu - 2\,\sigma$ < *95,5%* obs. < $\mu + 2\,\sigma$.
$\mu - 3\,\sigma$ < *99,7%* obs. < $\mu + 3\,\sigma$.
$\mu - 4\,\sigma$ < *+99,9%* obs. < $\mu + 4\,\sigma$.

The cumulative distribution function

• The last property implies that we can relate all normal random variables to the standard normal and inversely

• if Z is a standard normal distribution: $Z \sim N(0,1)$

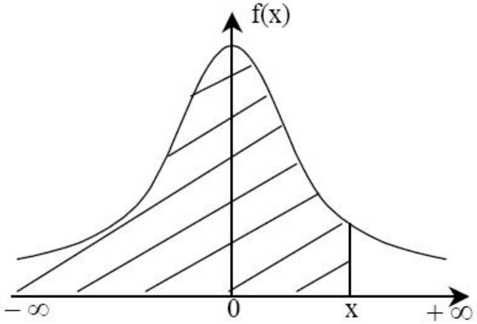
• $X = Z\sigma + \mu$

• We can deduce:

- the probability to observe either a smaller (lower tail) or a higher (upper tail) value than X*, and inversely
- the minimum value of X with a specified level of probability

The Gaussian Distribution

(Probability to find a value inferior to X)



$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du$$

X	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2,1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890
2,3	0,9893	0,9896	0,9898	0,9901	0,9904	0,9906	0,9909	0,9911	0,9913	0,9916
2,4	0,9918	0,9920	0,9922	0,9925	0,9927	0,9929	0,9931	0,9932	0,9934	0,9936
2,5	0,9938	0,9940	0,9941	0,9943	0,9945	0,9946	0,9948	0,9949	0,9951	0,9952
2,6	0,9953	0,9955	0,9956	0,9957	0,9959	0,9960	0,9961	0,9962	0,9963	0,9964
2,7	0,9965	0,9966	0,9967	0,9968	0,9969	0,9970	0,9971	0,9972	0,9973	0,9974
2,8	0,9974	0,9975	0,9976	0,9977	0,9977	0,9978	0,9979	0,9979	0,9980	0,9981
2,9	0,9981	0,9982	0,9982	0,9983	0,9984	0,9984	0,9985	0,9985	0,9986	0,9986
3,0	0,9987	0,9987	0,9987	0,9988	0,9988	0,9989	0,9989	0,9989	0,9990	0,9990
3,1	0,9990	0,9991	0,9991	0,9991	0,9992	0,9992	0,9992	0,9992	0,9993	0,9993
3,2	0,9993	0,9993	0,9994	0,9994	0,9994	0,9994	0,9994	0,9995	0,9995	0,9995
3,3	0,9995	0,9995	0,9995	0,9996	0,9996	0,9996	0,9996	0,9996	0,9996	0,9997
3,4	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9998
3,5	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998

Example with psychological and biological data

- IQ (mean=100; SD=15)
- French woman size (mean=162; SD=6.5) in 2001
- French men size (mean=174;SD=7.1) in 2001

1) What is the probability to find a woman with a size inferior to 174?

$Z = (174-162)/6.5 = 1.85$ then $P(sw<174) = P(Z<1.85) =$ **96.78%**

2) What is the probability to find a woman with a size inferior to 150?
$P(ws<150) = P(Z<-1.85) = 1-P(Z<1,85) = 1- 0.96786 =$ **3.24%**

3) What is the lowest size of 95% of women (denoted Y):
$P(ws>Y) = 0.95$; we know $P(Z<\underline{1.65}) = 95\%$ i.e. $P(Z>-\underline{1.65}) = 95\%$
$Y = -1.65 \times 6.5 + 162 =$ **151.31**

Using Excel

1) What is the probability to find a woman with a size inferior to 174?

=LOI.NORMALE(174;162;6,5;1)

2) What is the probability to find a woman with a size inferior to 150?
=LOI.NORMALE(150;162;6,5;1)

3) What is the lowest size of 95% of women (denoted Y):
=LOI.NORMALE.INVERSE(5%;162;6,5)

An example with financial data

- You invest your wealth in an European Equities Fund
- The annual mean return is: 9%
- The volatility of the fund measured by the standard error is 10%
- If we suppose than returns are normally distributed:

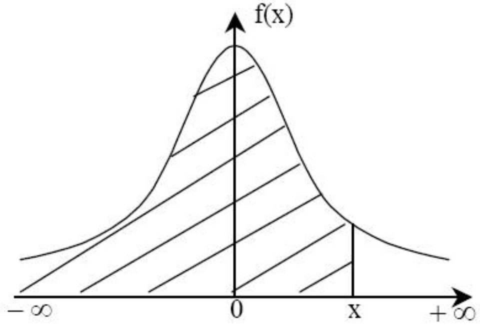
95,5% of chance that the annual return is between -11% and 29%

The worst return in 95% of cases: -7,5%

9% - 1.65 x 10% = -7,5%

The Gaussian Distribution

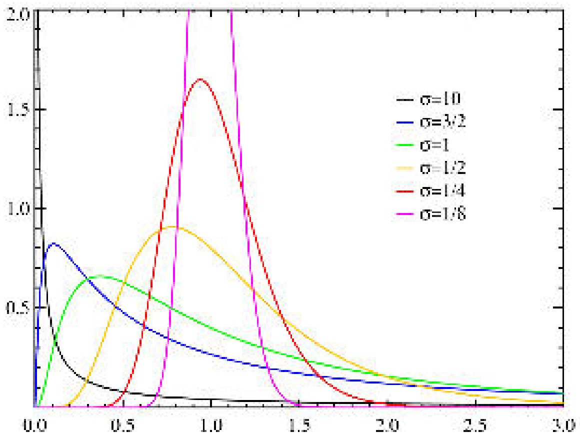
(Probability to find a value inferior to X)



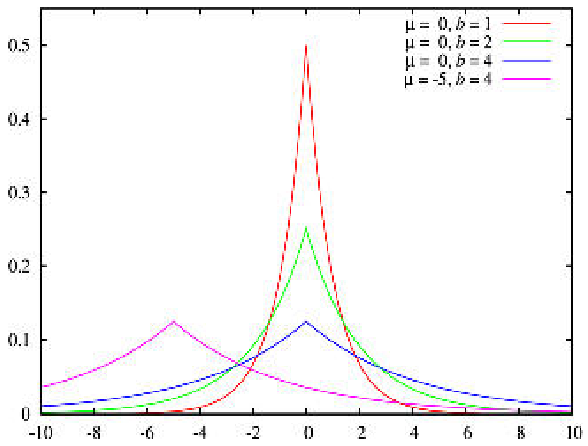
$$F(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}du$$

X	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2,1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890
2,3	0,9893	0,9896	0,9898	0,9901	0,9904	0,9906	0,9909	0,9911	0,9913	0,9916
2,4	0,9918	0,9920	0,9922	0,9925	0,9927	0,9929	0,9931	0,9932	0,9934	0,9936
2,5	0,9938	0,9940	0,9941	0,9943	0,9945	0,9946	0,9948	0,9949	0,9951	0,9952
2,6	0,9953	0,9955	0,9956	0,9957	0,9959	0,9960	0,9961	0,9962	0,9963	0,9964
2,7	0,9965	0,9966	0,9967	0,9968	0,9969	0,9970	0,9971	0,9972	0,9973	0,9974
2,8	0,9974	0,9975	0,9976	0,9977	0,9977	0,9978	0,9979	0,9979	0,9980	0,9981
2,9	0,9981	0,9982	0,9982	0,9983	0,9984	0,9984	0,9985	0,9985	0,9986	0,9986
3,0	0,9987	0,9987	0,9987	0,9988	0,9988	0,9989	0,9989	0,9989	0,9990	0,9990
3,1	0,9990	0,9991	0,9991	0,9991	0,9992	0,9992	0,9992	0,9992	0,9993	0,9993
3,2	0,9993	0,9993	0,9994	0,9994	0,9994	0,9994	0,9994	0,9995	0,9995	0,9995
3,3	0,9995	0,9995	0,9995	0,9996	0,9996	0,9996	0,9996	0,9996	0,9996	0,9997
3,4	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9997	0,9998
3,5	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998	0,9998

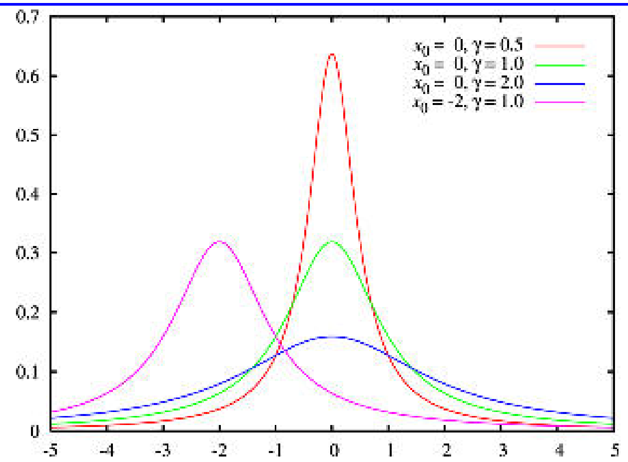
Some other distributions



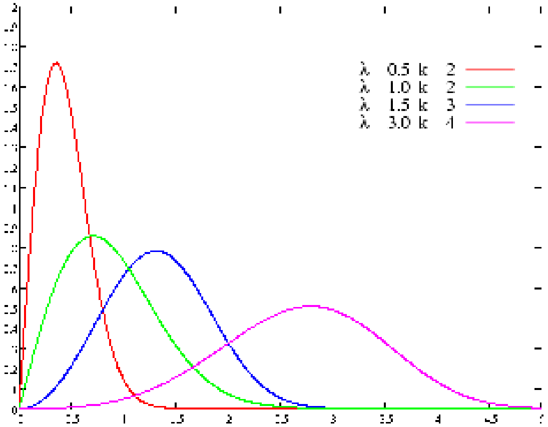
lognormal



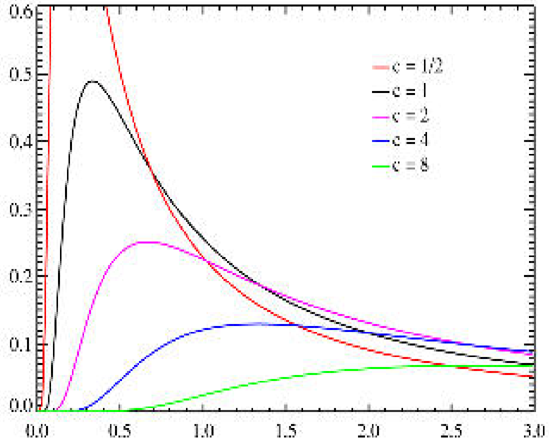
Laplace



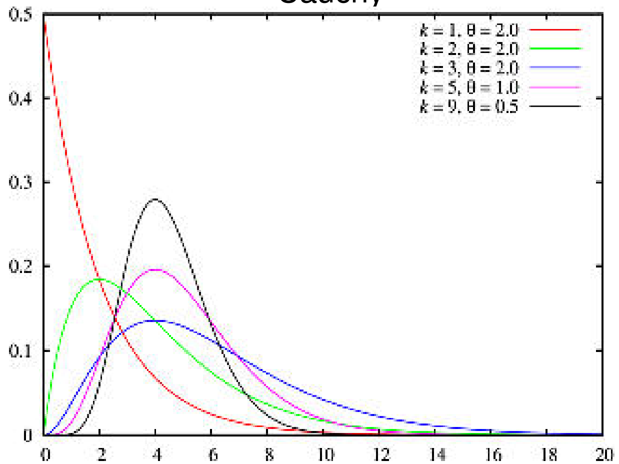
Cauchy



Weibull



Levy



Gamma

Observed financial data

- Skewness ≠ 0 and Kurtosis ≠ 3

$$Sk = \frac{E\left[R - E(R)\right]^3}{\boldsymbol{\sigma}(R)^3} \qquad Sk = \frac{\left(\frac{1}{n-1}\right)\sum_{t=1}^{n}(R_t - \boldsymbol{\mu})^3}{\boldsymbol{\sigma}(R)^3}$$

$$K = \frac{E\left[R - E(R)\right]^4}{\boldsymbol{\sigma}(R)^4} \qquad K = \frac{\left(\frac{1}{n-1}\right)\sum_{t=1}^{n}(R_t - \boldsymbol{\mu})^4}{\boldsymbol{\sigma}(R)^4}$$

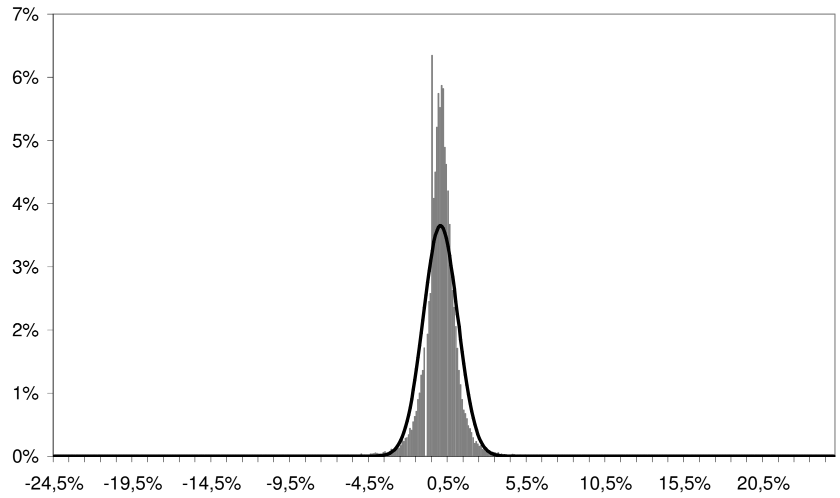
- Test normality e.g. with the Jarque-Bera test

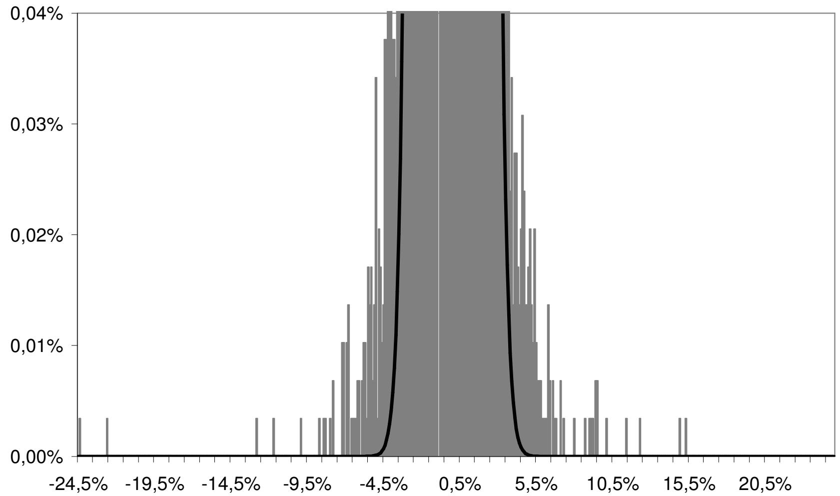
$$JB = \frac{n-2}{6}\left(Sk^2 + \frac{1}{4}(K-3)^2\right)$$

JB is distributed as Chi-Square with 2 degrees of. freedom

DJIA daily returns since 1896

Mean	0.000245
Median	0.000455
Maximum	0.153418
Minimum	-0.243909
Std. Dev.	0.010925
Skewness	-0.598300
Kurtosis	28.84078
Jarque-Bera	820579.5
Probability	0.000000

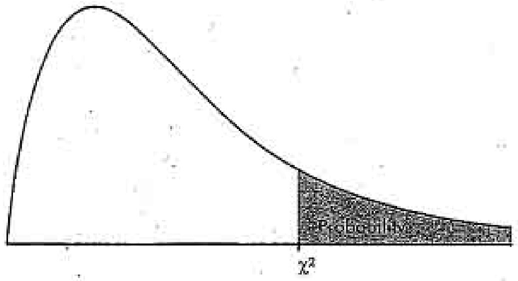




For large sample sizes, the test statistic has a chi-square distribution with two degrees of freedom.

The P value or calculated probability is the probability of wrongly rejecting the null hypothesis if it is in fact true.

Chi-square table



	Tail probability p										
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4

DJIA risk and “normal” risk

		Gaussian Law		Empirical Law	
Daily return (R*)		P(R<R*)	Frequency	P(R<R*)	Frequency
μ - 1 σ	-1,05%	15,87%	6 days	12,11%	8 days
μ - 2 σ	-2,14%	2,28%	43 days	2,85%	35 days
μ - 3 σ	-3,23%	0,14%	3 years	0,93%	4 months
μ - 4 σ	-4,32%	0,0032%	126 years	0,38%	1 year
μ - 6σ	-7,62%	1,29^{E-9} %	3 million years	0,04%	10 years

Source: Datastream and Dow Jones

4.2 Volatility: Traditional Measure of Risk

Standard Approach to Estimating Volatility

- Define σ_t as the volatility over N days
- Define P_t as the value of an asset at end of day t (closing prices)

- Define $r_t = \ln(P_t / P_{t-1})$

$$\sigma_t = \left[\frac{1}{(N-1)} \sum_{i=t-N}^{t} \left[r_i - \overline{r}_t \right]^2 \right]^{\frac{1}{2}}$$

$$\overline{r}_t = \frac{1}{N} \sum_{i=t-N}^{t} r_i$$

Simplifications usually made

- Define r_t as $(P_t - P_{t\text{-}1})/P_{t\text{-}1}$
- Assume that the mean value of r_t is zero
- Replace $N-1$ by N
- This gives:

$$\sigma_t = \left[\frac{1}{N} \sum_{i=t-N+1}^{t} r_i^2 \right]^{\frac{1}{2}}$$

- We use generally annualized volatility: $\sigma^a{}_t = \left[\frac{252}{N} \sum_{i=t-N+1}^{t} r_i^2 \right]^{\frac{1}{2}}$

Volatility drawbacks

- Advantages of volatility: well known concept in statistics, easily computed and easily interpretable (dispersion around mean returns)

- Asymmetrical distribution: positive and negative deviations from the average equally considered

- Investors are often more adverse to negative deviations and probability of extreme low returns

4.3 Alternative Risk Measures

Few alternative measures of risk

- High-Low Parkinson Measure
- Range
- MAD
- Probability of loss
- Semi-variance or downside variance
- VaR
- Expected Shortfall
- Worst loss
- Drawdown (amount, length, recovery time)

Parkinson (1980)

- High Low Estimator (high and low of the day *t):*

$$\sigma_t^{Park} = \left[\frac{250}{4\ln(2)N} \sum_{i=t-N+1}^{t} \left[\ln\left(H_i / L_i \right) \right]^2 \right]^{\frac{1}{2}}$$

- Use information about the range of prices in each trading day

See also…

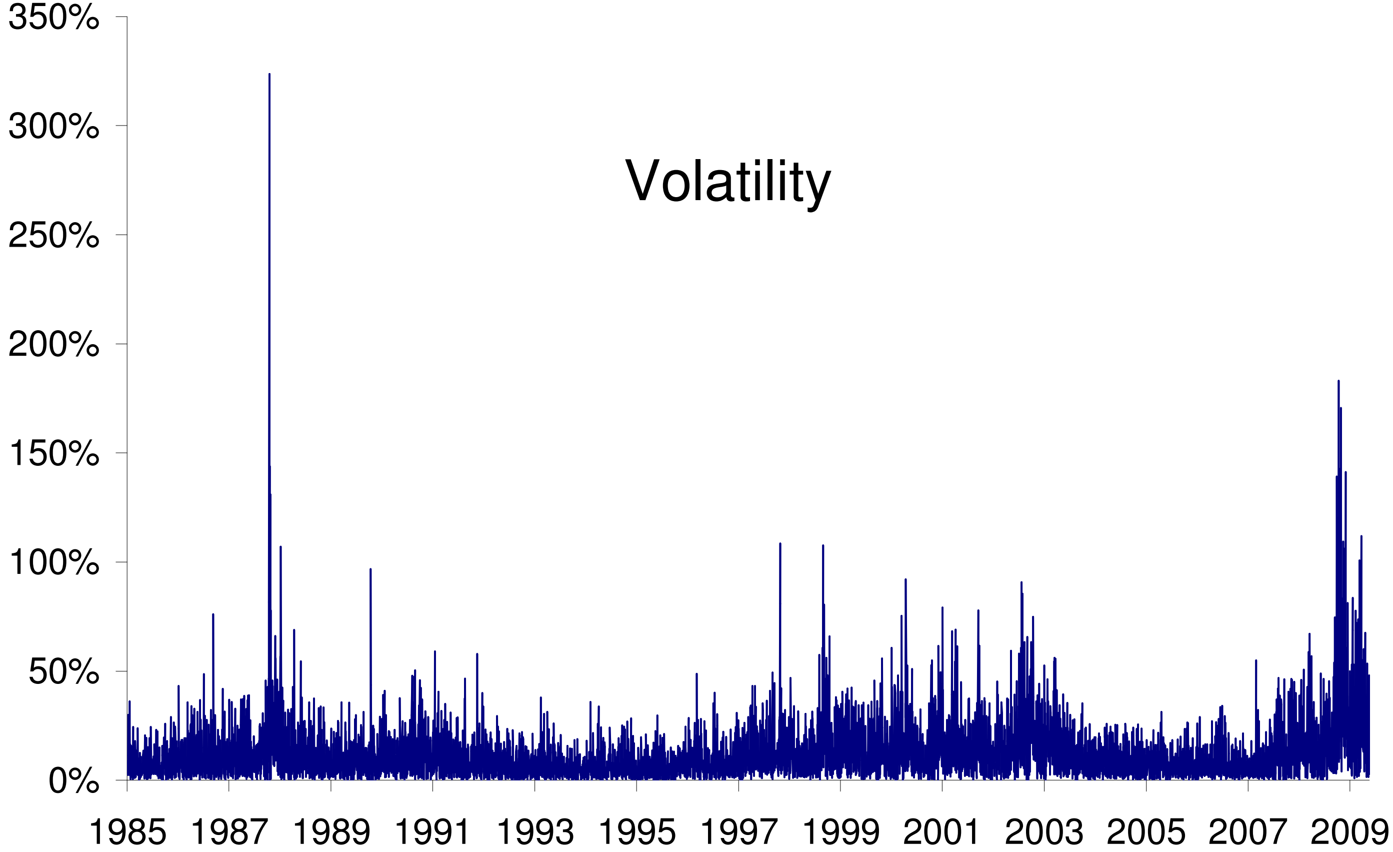
- Yang-Zhang (2000) Measure:

$$\sigma_t^{YZ} = \left[\frac{252}{N} \sum_{i=t-N+1}^{t} \left[\left(\ln \frac{O_i}{C_{i-1}} \right)^2 + \frac{1}{2} \left(\ln \frac{H_i}{L_i} \right)^2 - (2\ln 2 - 1) \left(\ln \frac{C_i}{O_i} \right)^2 \right] \right]^{\frac{1}{2}}$$

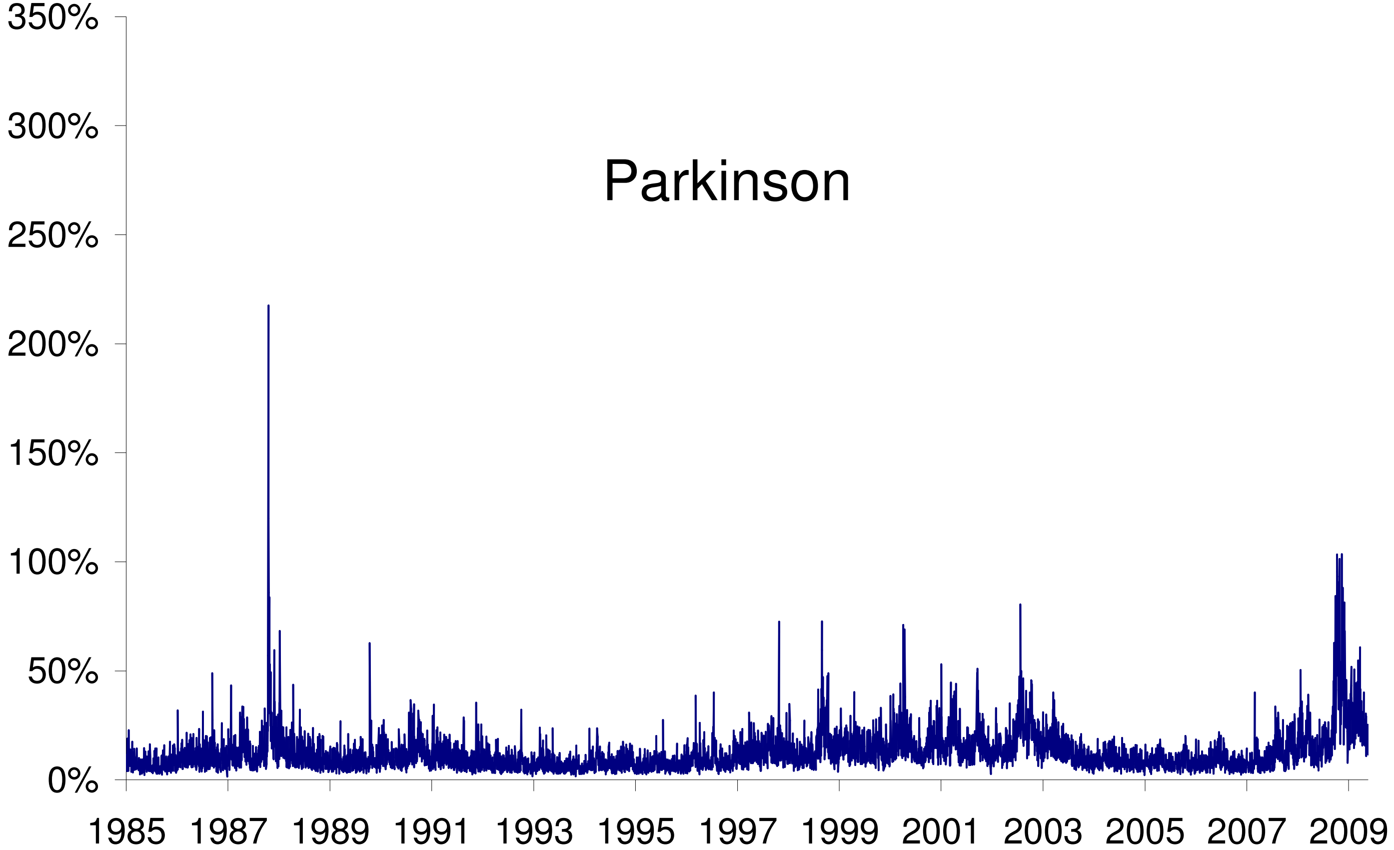
- Garman-Klass (1980) Measure:

$$\sigma_t^{GK} = \left[\frac{252}{N} \sum_{i=t-N+1}^{t} \left[\frac{1}{2} \left(\ln \frac{H_i}{L_i} \right)^2 - (2\ln 2 - 1) \left(\ln \frac{C_i}{O_i} \right)^2 \right] \right]^{\frac{1}{2}}$$

Traditional volatility and Parkinson measure on the S&P 500



Traditional volatility and Parkinson measure on the S&P 500



Range, MAD and probability of negative returns

• **Range**:
- Largest minus smallest Holding Period Return
- Misses full nature of distribution (variance, skewness)
- Can not determine statistically the range of mixing 2 or more securities

• **Mean Absolute Deviation**:
- Calculated in a similar manner as standard deviation, except you subtract the median and instead of squaring each deviation, you take the absolute value of each deviation.

• **Probability of negative return**:
- Percentage of time the Holding Period Return is negative.
- Same type of problems as with range
- Sometimes calculated as probability of an HPR lower than T-Bills

Downside risk, VaR and expected shortfall

• **Semi-variance and Downside risk**:

$$\sigma_t = \left[\frac{1}{(N-1)} \sum_{i=t-N}^{t} \left[r_i^- - r^* \right]^2 \right]^{\frac{1}{2}} \quad r_i^- = \begin{cases} r_i \text{ if } r_i < r^* \\ r^* \text{ if } r_i \geq r^* \end{cases} \qquad \text{and } r^* = \text{some target (e.g. } \overline{r}\text{)}$$

• **VaR**:

- Measures the worst loss to be expected of a portfolio over a given time horizon at a given confidence level

• **Expected Shortfall:**

- The "expected shortfall at q% level" is the expected return on the portfolio in the worst q% of the cases

Worst loss and drawdown statistics

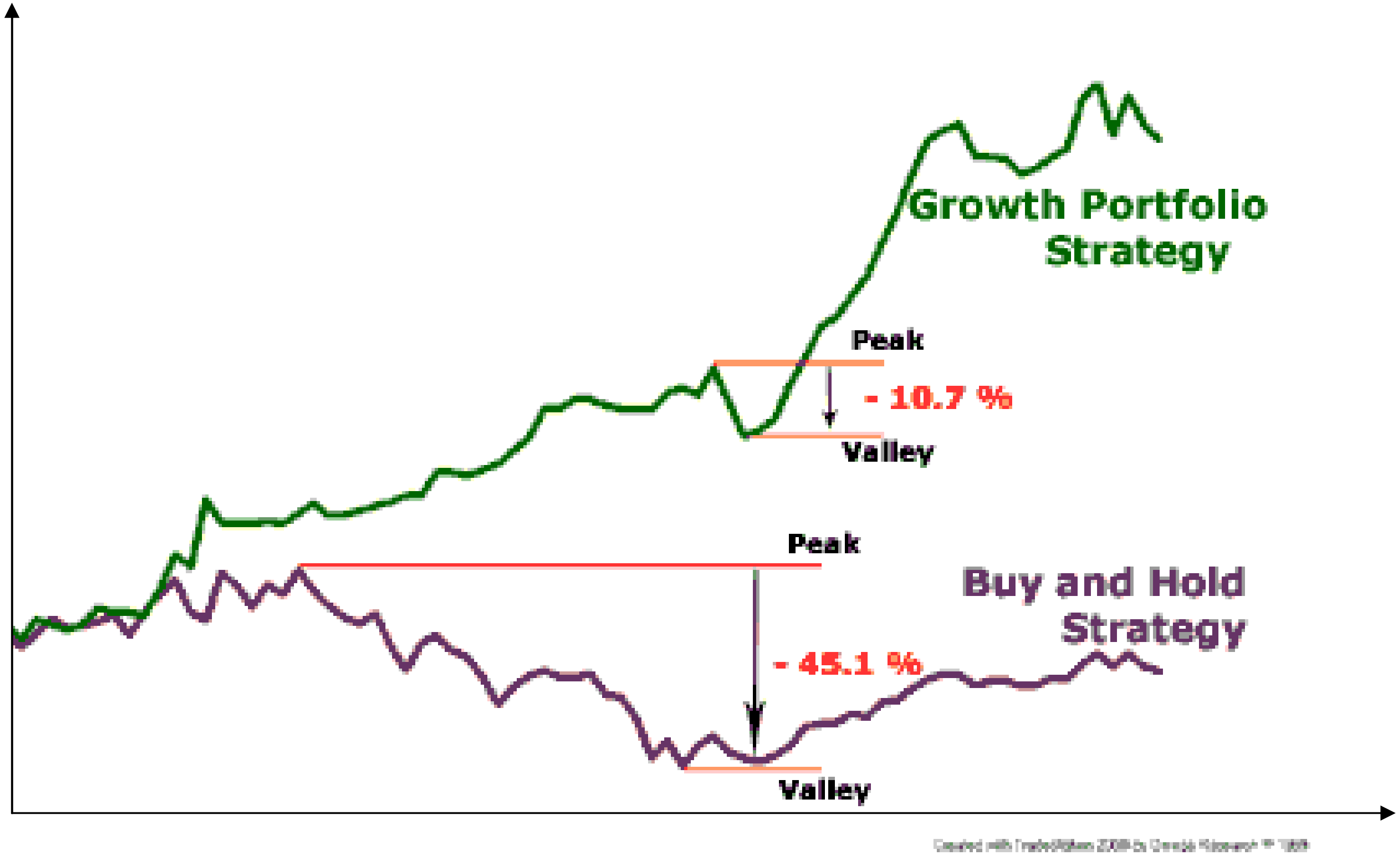
• **Worst loss**:
- Minimum return over a time period

• **Drawdown**:
- Maximum Drawdown measures losses experienced by a portfolio or asset over a specified period (worst loss over a period)

- Length: Peak-to-valley time period

- Recovery time: The time period from the previous peak to a new peak (New high ground).

Maximum Drawdowns



4.4 Lower Partial Moments

Lower partial moments

• **LPM**:

- The lower partial moment is the sum of the weighted deviations of each potential outcome from a pre-specified threshold level (r*), where each deviation is then raised to some exponential power (*n*).

- Like the semi-variance, lower partial moments are asymmetric risk measures in that they consider information for only a portion of the return distribution.

- Generalization of the semi-variance and shortfall risk

- The parameter n can be considered as a measure of risk aversion of the investor. If a shortfall is of serious concern, then a higher value of n can be used to capture that

$$LPM_n = \sum_{\tilde{R}_P = -\infty}^{r^*} p_p (r^* - \tilde{R}_P)^n = \sum_{p=1}^{K} p_p \left[\min(0, \tilde{R}_p - r^*) \right]^n$$

Example of Downside Risk Measures

Potential Return	Prob. of Return for Portfolio #1	Prob. of Return for Portfolio #2
-15%	5%	0%
-10	8	0
-5	12	25
0	16	35
5	18	10
10	16	7
15	12	9
20	8	5
25	5	3
30	0	3
35	0	3

Notice that the expected return for both of these portfolios is 5%:

$$E(R)_1 = (.05)(-0.15) + (.08)(-0.10) + ...+ (.05)(0.25) = 0.05$$

and

$$E(R)_2 = (.25)(-0.05) + (.35)(0.00) + ...+ (.03)(0.35) = 0.05$$

Example of Downside Risk Measures (cnt'd)

Variance

$$(Var)_1 = (.05)[-0.15 - 0.05]^2 + (.08)[-0.10 - 0.05]^2 + ... + (.05)[0.25 - 0.05]^2 = 0.0108$$

and

$$(Var)_2 = (.25)[-0.05 - 0.05]^2 + (.35)[0.00 - 0.05]^2 + ... + (.03)[0.35 - 0.05]^2 = 0.0114$$

Taking the square roots of these values leaves:

$$SD_1 = 10.39\% \quad \text{and} \quad SD_2 = 10.65\%$$

Example of Downside Risk Measures (cnt'd)

Semi-Variance

$$(\text{SemiVar})_1 = (.05)[-0.15 - 0.05]^2 + (.08)[-0.10 - 0.05]^2 + (.12)[-0.05 - 0.05]^2 + (.16)[0.00 - 0.05]^2 = 0.0054$$

and

$$(\text{SemiVar})_2 = (.25)[-0.05 - 0.05]^2 + (.35)[0.00 - 0.05]^2 = 0.0034$$

Also, the semi-standard deviations can be derived as the square roots of these values:

$$(\text{SemiSD})_1 = 7.35\% \quad \text{and} \quad (\text{SemiSD})_2 = 5.81\%$$

Example of Downside Risk Measures (cnt'd)

LPM1

$$(LPM1)_1 = (.05)[-0.00 + (-0.15)] + (.08)[-0.00 + (-0.10)] + (.12)[-0.00 + (-0.05)] = -0.0215$$

and

$$(LPM1)_2 = (.25)[-0.00 + (-0.05)] = -0.0125$$

LPM2

$$(LPM2)_1 = (.05)[-0.00 +(-0.15)]^2 + (.08)[-0.00 + (-0.10)]^2 + (.12)[-0.00 + (-0.05)]^2 = 0.0022$$

and

$$(LPM2)_2 = (.25)[-0.00 + (-0.05)]^2 = 0.0006$$

4.5 VaR and the Expected Shortfall

Value at Risk

- **VaR addresses both Probability and Exposure**

VaR is an estimate of the maximum possible loss for a pre-set confidence level over a specified time interval

- Pre-set confidence = probability
- Maximum possible loss = exposure

- **Example**

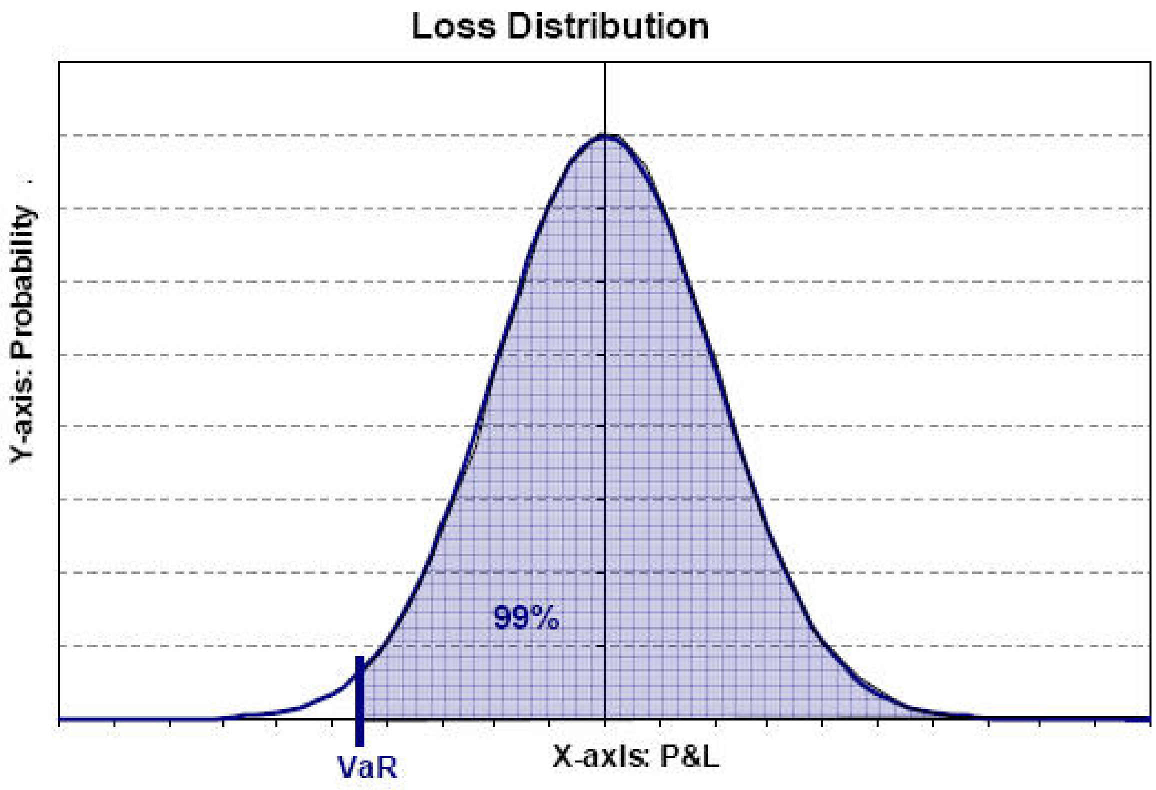
1-day 99% confidence level VaR is $140million

- 99% chance of losing less than $140 million over the next day, or
- 1% chance of losing more than $140 million over the next day

- **In terms of probability theory:**

VaR at the p% confidence level is the (1 - p)% quantile of the profit and loss distribution.

Illustration of VaR

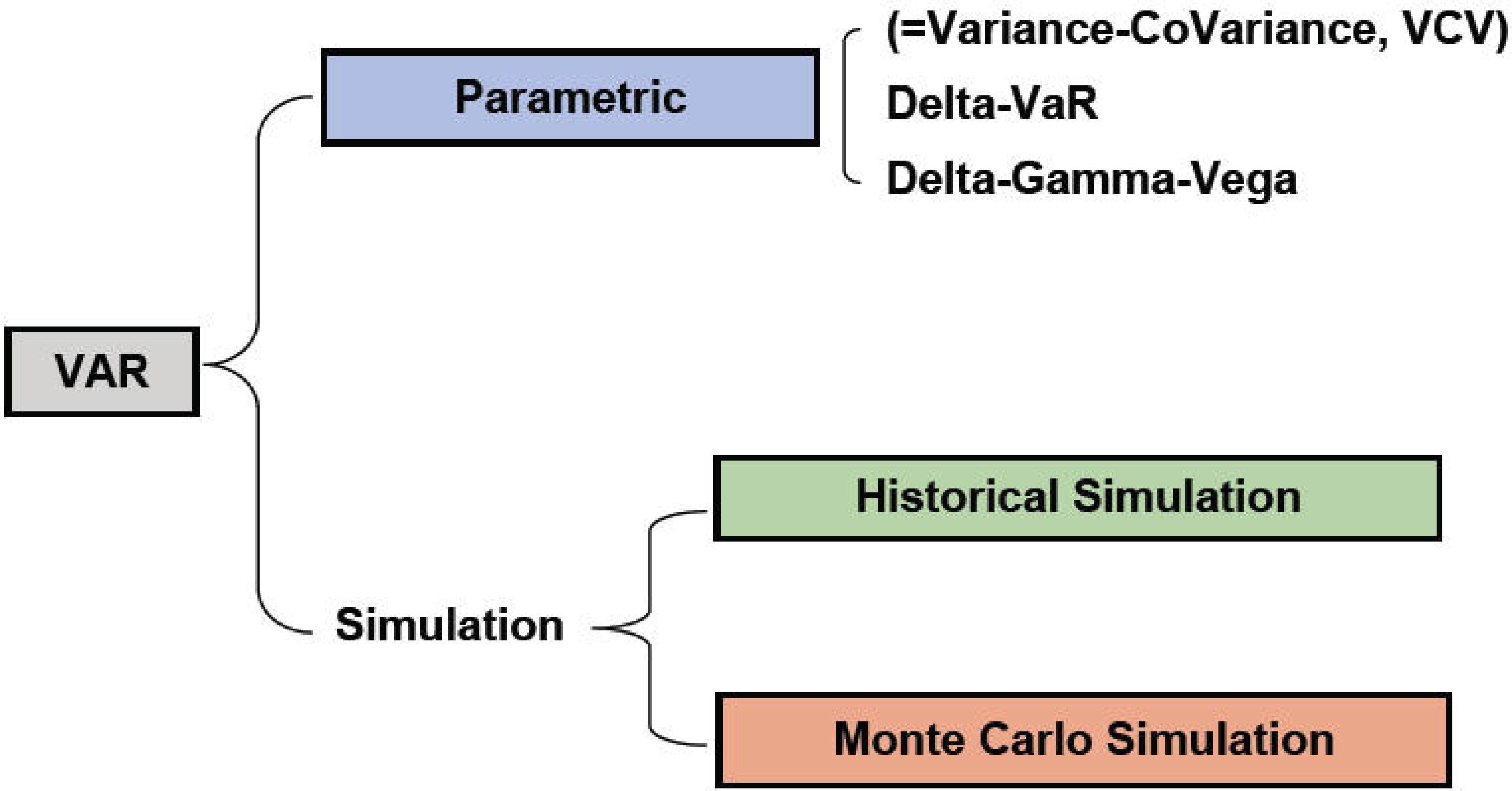


Distribution of possible P&L is crucial in calculating VaR

Value at Risk

- Conventional VaR
  - Simplicity of expression appeals to non-mathematicians (business types, regulators, marketeers)
  - A fractile of the return distribution rather than a dispersion measure
  - Consistent interpretation regardless of shape of return distribution
  - May be applied across asset classes and for all new types of securities
  - Easy to calculate
    - 3-4 methods
  - Cannot be optimised (not sub-additive or convex)

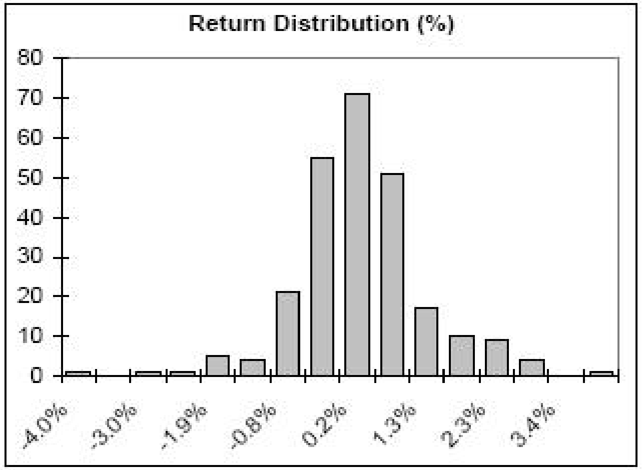
VaR Methodologies



Historical Simulation Example (1)

- Step 1. Distribution of 1 year historical return of stock XYZ

XYZ Bank	Close Price	Daily Return
6/1/2006	49.83	
6/2/2006	50.14	0.62%
6/5/2006	49.62	-1.04%
6/6/2006	49.76	0.28%
6/7/2006	49.95	0.38%
6/8/2006	49.95	0.00%
⋮	⋮	⋮
5/22/2007	55.08	0.44%
5/23/2007	55.01	-0.13%
5/24/2007	54.93	-0.15%
5/25/2007	55.12	0.35%
5/29/2007	54.91	-0.38%
5/30/2007	55.2	0.53%
5/31/2007	54.49	-1.29%
6/1/2007	54.51	0.04%
6/4/2007	54.15	-0.66%



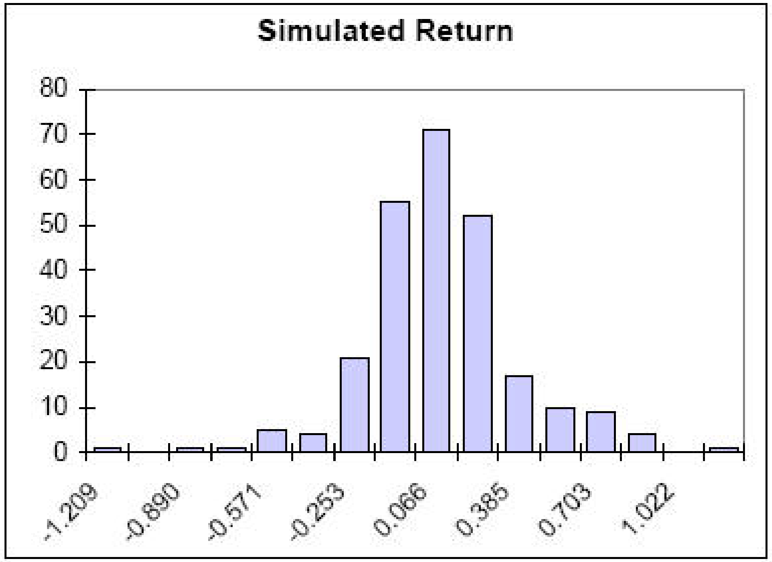
- Step 2. Simulate return of today's position (P_0): P_0 x return

  $30 x return

Historical Simulation Example (2)

- Step 3. Rank simulated return

P/L	Frequency
-1.209	1
-1.049	0
-0.890	1
-0.731	1
-0.571	5
-0.412	4
-0.253	21
-0.093	55
0.066	71
0.225	52
0.385	17
0.544	10
0.703	9
0.863	4
1.022	0
More	1
Total	252



- Step 4. Rank simulated returns

Take the average of the 2nd and 3rd worst return to estimate 99% VaR out of 252 data

1 day VaR at 99% confidence level is $.810

Parametric VaR Example (Returns)

- Step 1. Assume returns follow a normal distribution

- Step 2. Calculate mean returns (μ) and the standard errors (σ)

- Step 3. Using the 99th percentile 2.33 standard deviations,

1-day 99% VaR ≈ $P_0 \times (\mu - 2.33 \times \sigma)$

VaR Methodologies: General Calculation Steps

Parametric	Monte Carlo	Historical
1. Assume distribution for portfolio returns 2. Compute the present value of the portfolio 3. Compute the change in the portfolio value due to changes in the risk factor (risk factor volatility) 4. Apply the distributional assumptions at confidence level	1. Generate scenarios randomly 2. Revaluate portfolio under scenarios 3. Rank simulated P/L 4. Select n-th worst P/L at desired percentile	1. Calculate historical day-to-day changes in market rates/prices 2. Apply each change to today's market price (Revaluate) 3. Rank simulated losses from the least to greatest 4. Select n-th worst P/L at desired percentile

Comparing VaR Methodologies

	Parametric	Monte Carlo	Historical
Speed of computation	Fastest	Slow but could be optimized	Relatively Fast
Ability to capture non-linearity	Captures poorly if delta only	Could capture properly	Captures properly
Ability to capture non-normality	Captures poorly	Could capture properly	Captures properly
Dependence on historical data	Use data to estimate distribution	Use data to estimate distribution	Sensitive to "bad days"
Parametric assumptions	Required	Not required	Not required
Ease of use	Could be easy	Most costly to implement	Easy to implement and communicate

In practice

• The objective should be to provide a reasonably accurate estimate of risk at a reasonable cost, i.e., tradeoff between speed and accuracy.

• Historical simulation approach is used most widely. Parametric approach and Monte Carlo simulation are used as well but seems less popular when compared to historical simulation approach.

• For asset risk management: VaR is commonly computed on returns directly

1-day 99% VaR ≈ $\mu - 2.33 \times \sigma$

• Average returns in the parametric method

- With normal returns, VaR is simply equal to the average return minus a multiple of the volatility

- For a confidence level of 99% (95%), VaR is equal to the average return minus 2.326 (1.655) times the standard deviation

Time Horizon

- Instead of calculating the 10-day, 99% VaR directly analysts usually calculate a 1-day 99% VaR and assume

$$\text{10-day VaR} = \sqrt{10} \times \text{1-day VaR} = 10^{0.5} \times \text{1-day VaR}$$

- This is exactly true when portfolio changes on successive days come from independent identically distributed normal distributions

- If returns exhibit some positive (momentum) or negative (mean-reverting) autocorrelation at various horizon: +/-0.5 (≠regulatory instances)

Improving and supplementing VaR

- Parametric approach to Semi-parametric approach

- Expand parameters from delta-only to delta-gamma-vega to incorporate non-linearity and non-normality (e.g Cornish-Fisher expansion)

- Historical simulation approach

- Use longer historical dataset, Weighting scheme

- VaR optimizations

- Use PCA (Principal Components Analysis) to reduce number of risk factors

- Stress Testing, Scenario Analysis

- Sensitivity Analysis

Semi-parametric VaR

- VaR corrected for non-normality of financial series (skew. & kurt.)

$$z_\alpha^{CF} = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)S + \frac{1}{24}(z_\alpha^3 - 3z_\alpha)(K-3) - \frac{1}{36}(2z_\alpha^3 - 5z)S^2$$

$$VaR_{\boldsymbol{\alpha}}^{CF} = \boldsymbol{\mu} + z_{\boldsymbol{\alpha}}^{CF}\,\hat{\boldsymbol{\sigma}}$$

- Example: Portfolio of international equities (annual data)

Mean = 15% ; SE = 30% ;Skew = 0 ; Kurt =4

$$VaR_{5\%} = 0.15 - 1.655(0.3) = -0.3465 \qquad VaR_{1\%} = 0.15 - 2.33(0.3) = -0.549$$

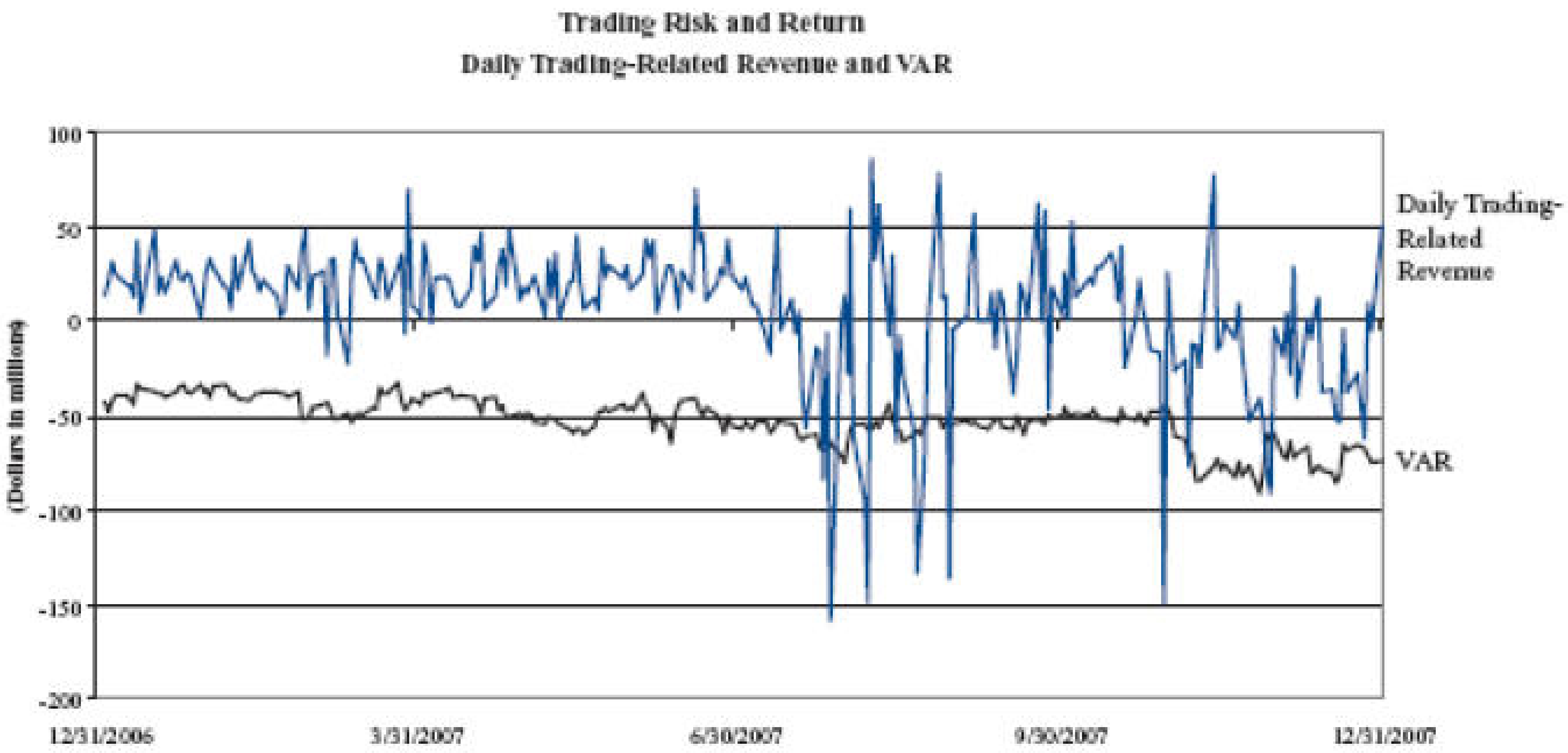
$$VaR_{5\%}^{CF} = 0.15 - 1.583(0.3) = -0.3249 \qquad VaR_{1\%}^{CF} = 0.15 - 3.273(0.3) = -0.8320$$

Backtesting

- Tests how well VaR estimates would have performed in the past

- We could ask the question: How often was the actual 1-day loss greater than the 99%/1 day VaR?

Backtesting example



From Bank of America's 2007 10K Report

Coherent risk measures (4 criteria)

- Translation invariance:
  – $\rho(X+\alpha) = \rho(X) - \alpha$ (for all α)

    If a non-risky investment of α is added to a risky portfolio, it will decrease the risk measure by α
- Sub-additivity:
  – $\rho(X+Y) \leq \rho(X)+\rho(Y)$

    The risk of o portfolio combining two sub-portfolios is smaller than the sum of the risk of the two sub-portfolios
- Monotonicity (more is better)
  – If $X \leq Y$ then $\rho(Y) \leq \rho(X)$

    If a portfolio X does better than portfolio Y under all scenarios, then the risk of X should be less than the risk of Y
- Positive homogeneity (scale invariance):
  – $\rho(\alpha X) = \alpha\, \rho(X)$ (for all $\alpha \geq 0$)

    If we increase the size of the portfolio by a factor α with the same weights, we increase the risk measure by the same factor α

VaR Drawbacks

- Simple and intuitive method of evaluating risk BUT:

- VaR is not a Coherent Risk Measure because they fail the third criteria

- the VaR of a portfolio with several components can be larger than the VaR of each of its components

- The quantile function is very unstable, un-robust at the tail

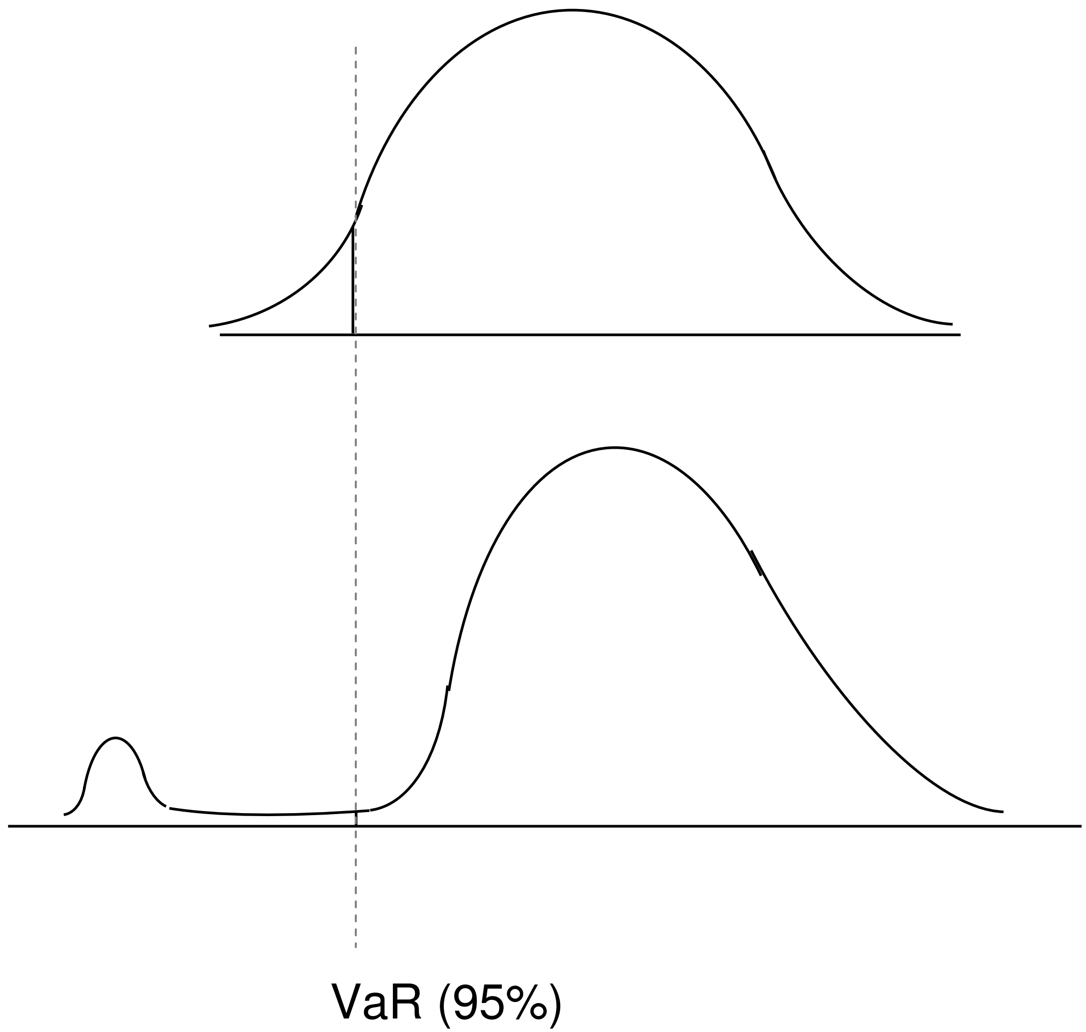
- VaR is highly sensitive to assumptions (manipulable)

- The actual extreme losses are unknown

- Gives only an upper limit on the losses given a confidence level tells nothing about the potential size of the loss if this upper limit is exceeded

"VaR gets me to 95% confidence.
I pay my Risk Managers to look after the remaining 5%"
- Dennis Weatherstone, former CEO, JP Morgan -

Different expected losses but same VaR

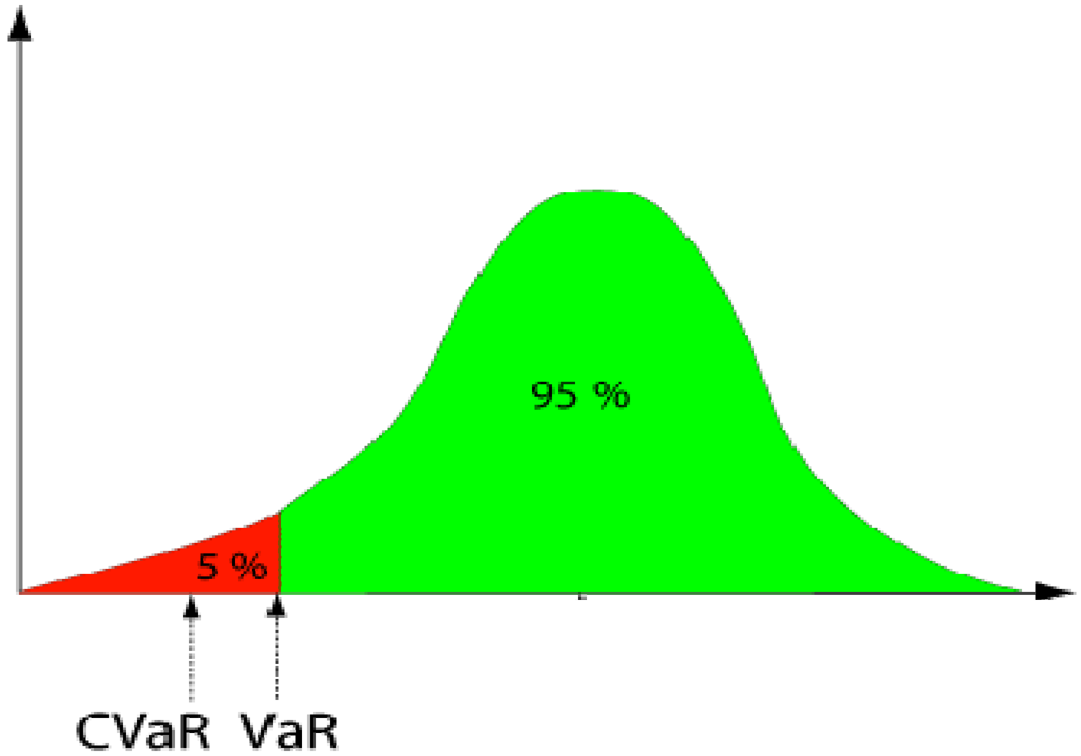


Expected shortfall or CVaR

- Two portfolios with the same VaR can have very different expected shortfalls
- Expected shortfall Measures the average loss to be expected of a portfolio over a given time horizon provided that VaR has been exceeded (also called C-VaR and Tail Loss)
- Maximal Loss is a limiting case of ES
- Both ML and ES are coherent measures

$$ES(\alpha) = E[r \mid r < VaR(\alpha)]$$

VaR and CVaR (Expected shortfall)



Advantages of CVaR

- Conditional VaR (Expected Shortfall)
  - Combination fractile and dispersion measure
  - One of a class of lower partial moment measures
  - Reveals the nature of the tail
  - More suitable than VaR for stress testing
  - Improvement on VaR in that CVaR is subadditive and coherent
  - When optimising, CVaR frontiers are properly convex
- This is a more robust downside measure than VaR
- Although CVaR is theoretically more appealing, it is not widely used

4.6 Geometric Mean and Safety First Criteria

Road map

- Geometric Mean Return criterion

- Safety first criteria
  - Roy
  - Kataoka
  - Telser

Geometric Mean Return Criterion

- **Select the portfolio that has the highest expected geometric mean return**
- **Proponents of the GMR portfolio argue:**

  - Has the highest probability of reaching, or exceeding, any given wealth level in the shortest period of time

  - Has the highest probability of exceeding any given wealth level over any period of time

- **Opponents argue that expected value of terminal wealth is not the same as maximizing the utility of terminal wealth:**

  - May select a portfolio not on the efficient frontier

Properties of the GMR Portfolio (1)

- **A diversified portfolio usually has the highest geometric mean return**

- **A strategy that has a possibility of bankruptcy would never be selected**

- **The GMR portfolio will generally not be mean-variance efficient unless:**
  - Investors have a log utility function and returns are normally or log-normally distributed

Properties of the GMR Portfolio (2)

- **Maximize the Geometric Mean Return**

1. Has the highest expected value of terminal wealth
2. Has the highest probability of exceeding given wealth level

What is Geometric Mean ?

$$\overline{R}_A = \left[(1+R_1)+(1+R_2)+(1+R_3)\right]\frac{1}{3} - 1.0$$

$$\overline{R}_A = \frac{1}{T}\sum_{t=1}^{T}(1+R_t) - 1.0$$

$$R_G = \left[(1+R_1)(1+R_2)(1+R_{3)}\right]^{1/3} - 1.0$$

$$R_G = \prod_{t=1}^{T}\left[(1+R_t)\right]^{1/T} - 1.0$$

Safety-first
- selection criteria

- **Objective**

- To limit the risk of unacceptable or unsatisfactory outturns

- **Three approaches**

- Roy – minimise risk
- Kataoka – set floor to out-turn
- Telser – select best performing portfolio if safety constraint is satisfied

Safety-first selection criteria

- Roy's criterion is to select a minimum return level and minimise the risk of failing to reach this.
- Kataoka's criterion is to specifically accept a risk level (safety constraint) and select the portfolio which offers the highest minimum return (floor) consistent with that risk level.
- Telser's criterion is to revert to the intuitively appealing selection criterion of highest average return but forcing adherence to a constraint which recognises both a risk level and a minimum desired return level (safety constraint).

Roy’s criterion (1)

- Best portfolio is the one that has the lowest probability of producing a return below the specified level

- Investor subjectively selects a lower acceptable return R_L below which she does not want the portfolio return falling

- Criterion objectively selects the preferred portfolio on the basis that it has the least chance of falling below R_L

Roy's criterion (2)

$$\text{Min Prob}(R_P < R_L)$$

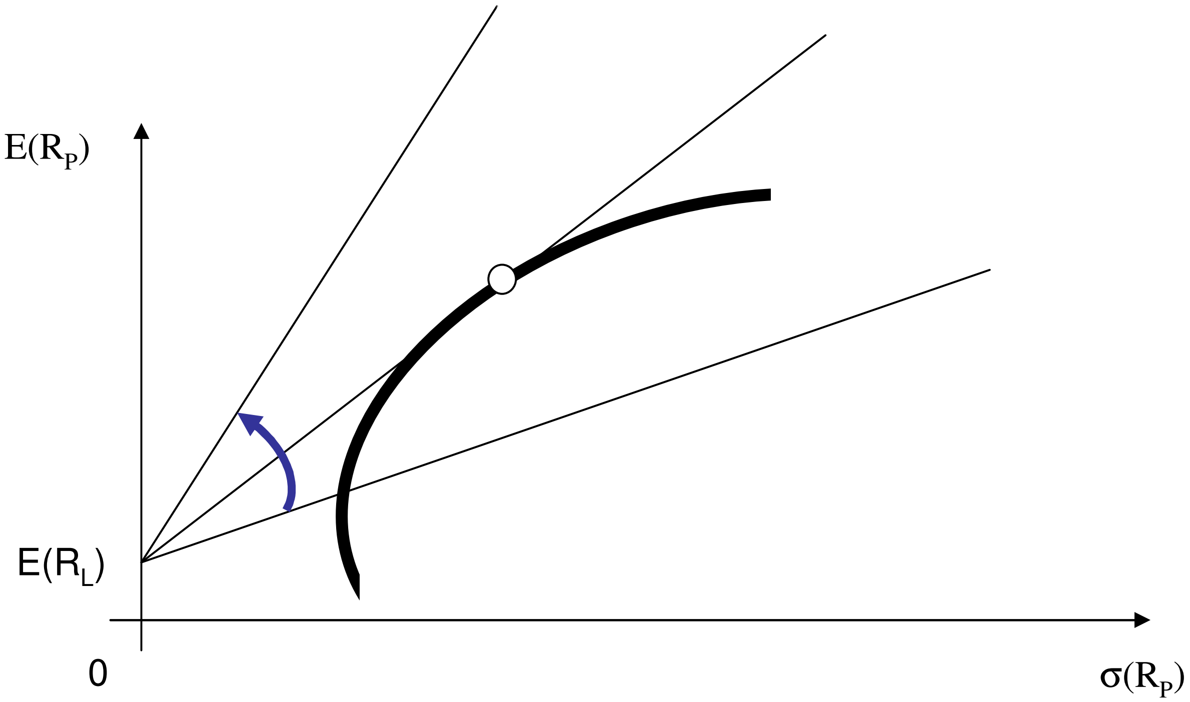
If normal distribution $\Rightarrow$ minimise the number of SE the mean is below the lower limit

$$\text{Min Prob}\left(\frac{R_P - \overline{R_P}}{\sigma_P} < \frac{R_L - \overline{R_P}}{\sigma_P}\right)$$

$$\Leftrightarrow \text{Min}\left(\frac{R_L - \overline{R_P}}{\sigma_P}\right)$$

$$\Leftrightarrow \text{Max}\left(\frac{\overline{R_P} - R_L}{\sigma_P}\right)$$

Roy's criterion (3)



Example

The risk free rate is 5% (JC Trichet is very anxious)

Consider the following three funds, what is the optimal portfolio under the Roy's criterion?

	A	B	C
Mean (%)	9	11	12
SD (%)	4	5	9

The optimal portfolio is the one that minimizes $\left(\frac{R_L - \overline{R_P}}{\sigma_P}\right)$

A	(5 – 9)/4 = -1
B	**(5 – 11)/5 = -1.2**
C	(5 – 12)/9 = -0.78

Kataoka's criterion (1)

- Maximize the lower limit (R_L) subject to the probability that the portfolio return being less than the lower limit, is not greater than some specified value.
- The investor subjectively selects an acceptable risk level α.
- The criterion then objectively selects the fund which produces the highest return floor.

Kataoka's criterion (2)

$$\text{Max}(R_L)$$

$$\text{u.c.} \quad \text{Prob}(\text{R}_\text{P} < R_L) \leq \alpha$$

If the returns are normally distributed:

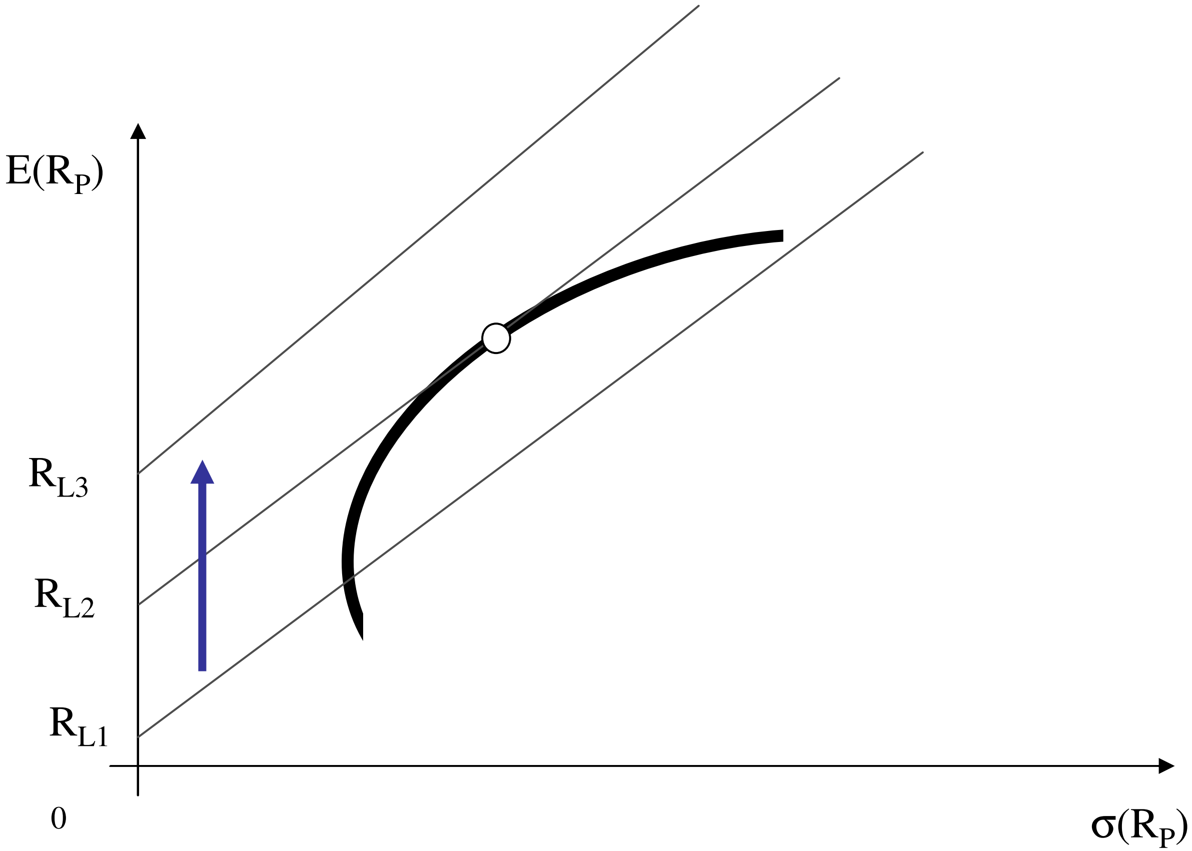
$$\Rightarrow \text{Prob}\left(\frac{\widetilde{R_P} - \overline{R_P}}{\sigma_P} < \frac{R_L - \overline{R_P}}{\sigma_P}\right) \leq \alpha$$

for α = 5%

$$\Rightarrow \frac{\overline{R_P} - R_L}{\sigma_P} = 1{,}65 \quad \Rightarrow \overline{R_P} = R_L + 1{,}65\sigma_P$$

Kataoka's criterion (2)

Then we maximize the intercept R_L of the straight line with a slope of 1.65



Telser's criterion (1)

- Select the portfolio with the highest average return, provided it obeys a safety constraint.

- Constraint is effectively a combination of Roy and Kataoka => that the probability of a return below some floor is not greater than some specified level.

- The investor makes two subjective inputs:

  - Choice of acceptable risk level $= \alpha$

  - Choice of acceptable lower limit $= R_L$

- The criterion then objectively selects the portfolio with the highest expected return satisfying the constraint.

Telser's criterion (2)

Maximize expected return subject to

The probability of a lower limit is no greater than some specified level

e.g., Maximize expected return given that the chance of having a negative return is no greater than 10%

$$\text{Max } \overline{R}_P$$

$$\text{u.c. Prob}(\tilde{R}_P < R_{Min}) \leq \boldsymbol{\alpha}$$

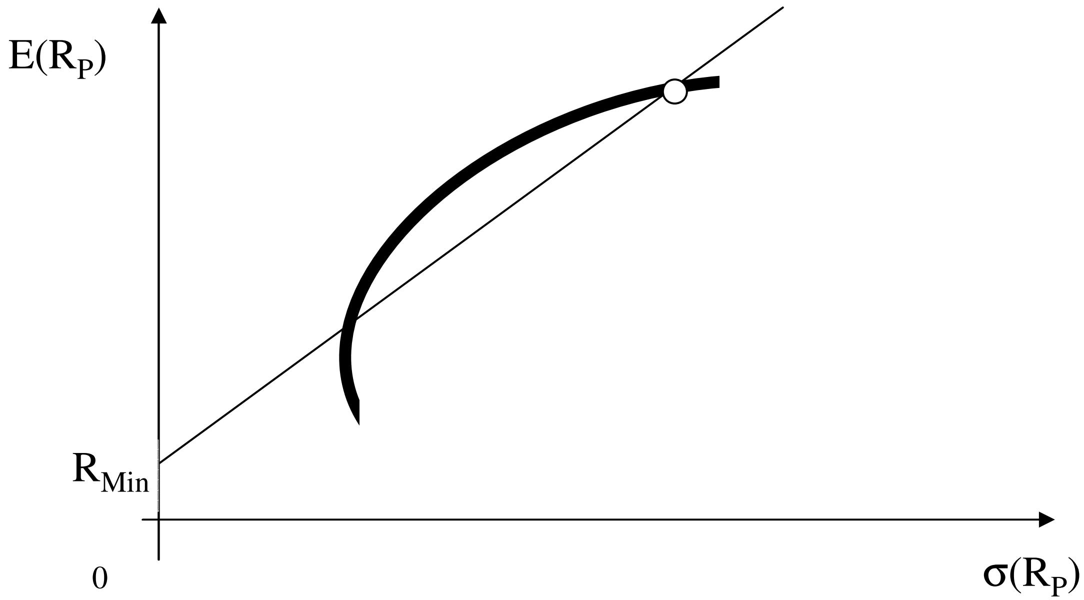
Under the normality assumption and for $\alpha = 5\%$, the constraint becomes:

$$\frac{\overline{R_P} - R_{Min}}{\boldsymbol{\sigma}_P} = 1,65 \quad \Rightarrow \quad \overline{R_P} = R_{Min} + 1,65\boldsymbol{\sigma}_P$$

Telser's criterion (3)

• The constraint effectively operates as a filter. Only portfolios meeting the constraint qualify for consideration

• The optimum portfolio either lies on the efficient frontier in mean standard deviation space or it does not exist.



Exercise

- Consider the following investments:

A		B		C	
Probability	Outcome	Probability	Outcome	Probability	Outcome
0,2	4%	0,1	5%	0,4	6%
0,3	6%	0,3	6%	0,3	7%
0,4	8%	0,2	7%	0,2	8%
0,1	10%	0,3	8%	0,1	10%
		0,1	9%		

Which of them do you prefer ?

1) Using first- and second-order stochastic dominance criteria ?
2) Using Roy's Safety first criterion if the minimum return is 5%
3) Using Kataoka's Safety first criterion if the probability is 10%
4) Using Telser's Safety first criterion with R_L = 5% and α = 10%
5) Using Geometric Mean Return criterion ?

FOSD and SOSD

	Cumulative Probability			Cumulative Cumulative Probability		
Outcome	A	B	C	A	B	C
4%	0,2	0,0	0,0	0,2	0,0	0,0
5%	0,2	0,1	0,0	0,4	0,1	0,0
6%	0,5	0,4	0,4	0,9	0,5	0,4
7%	0,5	0,6	0,7	1,4	1,1	1,1
8%	0,9	0,9	0,9	2,3	2,0	2,0
9%	0,9	1,0	0,9	3,2	3,0	2,9
10%	1,0	1,0	1,0	4,2	4,0	3,9

No investment exhibits first-order stochastic dominance.

	Cumulative Probability			Cumulative Cumulative Probability		
Outcome	A-B	B-C	A-C	A-B	B-C	A-C
4%	0,2	0,0	0,2	0,2	0,0	0,2
5%	0,1	0,1	0,2	0,3	0,1	0,4
6%	0,1	0,0	0,1	0,4	0,1	0,5
7%	-0,1	-0,1	-0,2	0,3	0,0	0,3
8%	0,0	0,0	0,0	0,3	0,0	0,3
9%	-0,1	0,1	0,0	0,2	0,1	0,3
10%	0,0	0,0	0,0	0,2	0,1	0,3

Using second-order stochastic dominance:
$C > B > A$

Roy's safety-first criterion

- Roy's safety-first criterion is to minimize Prob($R_P < R_L$). If $R_L = 5\%$, then

p =	A	B	C
P(Rp<5%)	0.2	**0.0**	**0.0**

Thus, using Roy's safety-first criterion, investments *B* and *C* are preferred over investment *A*, and the investor would be indifferent to choosing either investment *B* or *C*.

Kataoka's safety-first criterion

• Kataoka's safety-first criterion is to maximize R_L subject to $Prob(R_P < R_L) \leq \alpha$. If $\alpha = 10\%$, then :

	A	B	C
Max R_L	3.99%	5.99%	5.99%

Thus, B and C are preferred to A, but are indistinguishable from each other.

Telser's criterion

- Telser's criterion is to maximise Rp subject to the safety first constraint: Prob(R_P < 5%) ≤ 10%

p =	A	B	**C**
E(Rp)	6.80%	7.00%	**7.10%**
safety constraint	No	ok	**ok**

Employing Telser's criterion, we see that Project A does not satisfy the constraint Prob(R_p < 5%) ≤ 10%. Thus it is eliminated.

Between B and C, Project C has higher expected return (7.1% compared to 7%). Thus it is preferred.

GMR criterion

- The geometric mean returns of the investments are:

A		B		C	
Probability	Outcome	Probability	Outcome	Probability	Outcome
0.2	4%	0.1	5%	0.4	6%
0.3	6%	0.3	6%	0.3	7%
0.4	8%	0.2	7%	0.2	8%
0.1	10%	0.3	8%	0.1	10%
		0.1	9%		
6.78%		6.99%		**7.09%**	

$(1.04)^{.2}\ (1.06)^{.3}\ (1.08)^{.4}\ (1.1)^{.1} - 1 = .0678$ (6.78%)

$(1.05)^{.1}\ (1.06)^{.3}\ (1.07)^{.2}\ (1.08)^{.3}\ (1.09)^{.1} - 1 = .0699$ (6.99%)

$(1.06)^{.4}\ (1.07)^{.3}\ (1.08)^{.2}\ (1.1)^{.1} - 1 = .0709$ (7.09%).

Thus, C > B > A.

Risk measure application

- Consider the following days on the S&P 500:

	S&P500
20/10/2008	985.4
21/10/2008	955.05
22/10/2008	896.78
23/10/2008	908.11
24/10/2008	876.77
27/10/2008	848.92
28/10/2008	940.51
29/10/2008	930.09
30/10/2008	954.09
31/10/2008	968.75
03/11/2008	966.3

Using Excel calculate:

MEAN
MEDIAN
MIN
MAX
SE
Sk
Ku
HVaR 90%
HVaR 80%

Max DD
length
recovery
CVaR 79%
LPM1(MEAN)
LPM2(MEAN)
LPM1(0%)
LPM2(0%)
JB

Returns

	S&P500	Returns	Ordered returns	DD = $(P_t/MAX_{to,t})-1$
20/10/2008	985.4			0.00%
21/10/2008	955.05	-3.08%	-6.10%	-3.08%
22/10/2008	896.78	-6.10%	-3.45%	-8.99%
23/10/2008	908.11	1.26%	-3.18%	-7.84%
24/10/2008	876.77	-3.45%	-3.08%	-11.02%
27/10/2008	848.92	-3.18%	-1.11%	-13.85%
28/10/2008	940.51	10.79%	-0.25%	-4.56%
29/10/2008	930.09	-1.11%	1.26%	-5.61%
30/10/2008	954.09	2.58%	1.54%	-3.18%
31/10/2008	968.75	1.54%	2.58%	-1.69%
03/11/2008	966.3	-0.25%	10.79%	-1.94%

LPM calculations

$$\sum_{p=1}^{K} p_p \left[\min(0, \widetilde{R}_P - r^*) \right]^n$$

	n = 1	n = 2	n = 1	n = 2
	r*=MEAN	r*=MEAN	r*=0	r*=0
	-2,98%	0,09%	-3,08%	0,09%
	-6,00%	0,36%	-6,10%	0,37%
	0,00%	0,00%	0,00%	0,00%
	-3,35%	0,11%	-3,45%	0,12%
	-3,08%	0,09%	-3,18%	0,10%
	0,00%	0,00%	0,00%	0,00%
	-1,01%	0,01%	-1,11%	0,01%
	0,00%	0,00%	0,00%	0,00%
	0,00%	0,00%	0,00%	0,00%
	-0,15%	0,00%	-0,25%	0,00%
Σ	-1,66%	0,07%	-1,72%	0,07%

Risk measures

MEAN	-0.10%
MEDIAN	-0.68%
MIN	-6.10%
MAX	10.79%
SE	4.68%
Sk	1.37
Ku	5.82
HVaR 90%	-6.10%
HVaR 80%	-3.45%

Max DD	-13.85%
length	5
recovery	not yet
CVaR 79%	-4.78%
LPM1(MEAN)	-1.66%
LPM2(MEAN)	0.07%
LPM1(0%)	-1.72%
LPM2(0%)	0.07%
JB	5.14898864

Normality is not rejected at the 5% significance level (see slide 12)

Thank you for your attention…

See you next week