

Portfolio Choice

2009/2010

Session 8

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 *Variances*

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 ABN-AMRO



Part 8. Bond Prices and Management

8.1 Bond Prices and Yields

8.2 The Term-Structure of Interest Rates

8.3 Managing Bond Portfolios

8.1 Bond Prices and Yields

Bond Characteristics

- Face or par value
- Coupon rate
 - Zero coupon bond
- Compounding and payments
 - Accrued Interest
- Indenture



Different Issuers of Bonds

- U.S. Treasury
 - Notes and Bonds
- Corporations
- Municipalities
- International Governments and Corporations
- Innovative Bonds
 - Floaters and Inverse Floaters
 - Asset-Backed
 - Catastrophe



French Government Debt

- Agence France Tresor Mandate
 - Agence France Trésor manages central government debt and cash under the most secure conditions in the best interest of the taxpayer
- AFT Strategy:
 - In order to minimise debt costs over time, AFT sets priority to stability and predictability.
 - AFT does not aim at beating the market through opportunistic transactions.

French Government Debt (cnt'd)

Negotiable government debt and swaps

EUR billion

	end 2001	end 2002	end 2003	end 2004	end 2005	end 2006	end 2007	end of October 2006	end of November 2006
Negotiable government debt outstanding	653	717	768	833	877	877	921	996	1 013
OAT	443	478	512	552	593	610	641	672	681
BTAN	158	151	167	184	189	200	202	193	198
BTF	52	88	109	97	95	66	78	132	134

Source: AFT

French Government Debt (cnt'd)

- **Obligations assimilables du Trésor (OATs**, or fungible Treasury bonds):
LT bonds with maturities from seven to fifty years. Most OATs are fixed-rate bonds redeemable on maturity + floating-rate bonds (TEC 10 OATs pegged to the constant 10-year maturity rate) and inflation-indexed bonds (OAT_i, OAT_{€i}).
- **Bons du Trésor à intérêts annuels (BTANs** or negotiable fixed-rate medium-term Treasury notes with annual interest):
Medium-term government debt, maturity is either two or five years.
- **Bons du Trésor à taux fixe et à intérêts précomptés (BTFs** or negotiable fixed-rate discount Treasury bills):
The government's cash management instrument. They are used to cover short-term fluctuations in the government's cash position (less than one year – 4 to 52 weeks)
+ inflation-indexed bonds (BTAN_{€i}).

Listing of Treasury Issues

Treasury Bonds, Notes and Bills										January 17, 2006					
Explanatory Notes										MATURITY	TYPE	BID	ASKED	CHG	ASK YLD
<p>Representative Over-the-Counter quotation based on transactions of \$1 million or more. Treasury bond, note and bill quotes are as of mid-afternoon. Colons in bid-and-asked quotes represent 32nds; 101:01 means 101 1/32. Net changes in 32nds. n-Treasury note. I-Inflation-Indexed Issue. Treasury bill quotes in hundredths, quoted on terms of a rate of discount. Days to maturity calculated from settlement date. All yields are to maturity and based on the asked quote. Latest 13-week and 26-week bills are boldfaced. For bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par. *When issued.</p> <p>Source: eSpeed/Cantor Fitzgerald</p> <p>U.S. Treasury strips as of 3 p.m. Eastern time, also based on transactions of \$1 million or more. Colons in bid and asked quotes represent 32nds; 99:01 means 99 1/32. Net changes in 32nds. Yields calculated on the asked quotation. ci-stripped coupon interest bp-Treasury bond, stripped principal, np-Treasury note, stripped principal. For bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par.</p> <p>Source: Bear, Stearns & Co. via Street Software Technology Inc.</p>										Feb 06	bp	99:22	99:22	...	3.91
										Feb 06	np	99:22	99:23	...	3.86
										Apr 06	ci	99:00	99:01	...	4.09
										May 06	ci	98:21	98:22	...	4.15
										May 06	np	98:19	98:20	...	4.32
										May 06	np	98:19	98:20	...	4.32
										Jul 06	ci	98:03	98:03	1	3.94
										Aug 06	ci	97:19	97:20	1	4.23
										Aug 06	np	97:15	97:16	...	4.44
										Oct 06	ci	96:28	96:29	...	4.30
										Nov 06	ci	96:16	96:17	...	4.34
										Nov 06	np	96:14	96:14	...	4.45
										Nov 06	np	96:14	96:14	...	4.45
										Feb 07	ci	95:19	95:20	...	4.21
										Feb 07	np	95:13	95:14	...	4.39
										May 07	ci	94:18	94:18	...	4.26
										May 07	np	94:14	94:15	...	4.34
										May 07	np	94:14	94:15	...	4.35
										Aug 07	ci	93:18	93:18	...	4.26
										Aug 07	np	93:15	93:16	...	4.32
										Aug 07	np	93:14	93:15	...	4.33
										Nov 07	ci	92:17	92:18	...	4.29
										Nov 07	np	92:16	92:16	...	4.32
										Feb 08	ci	91:19	91:20	1	4.26
										Feb 08	np	91:19	91:19	1	4.27
										Feb 08	np	91:16	91:17	1	4.31
										May 08	ci	90:20	90:21	1	4.27
										May 08	np	90:18	90:19	1	4.30
										May 08	np	90:18	90:19	1	4.30
										Aug 08	ci	89:23	89:23	1	4.25
										Aug 08	np	89:21	89:21	1	4.28
										Nov 08	ci	88:22	88:23	1	4.29
										Feb 15	ci	67:02	67:03	4	4.45
										Feb 15	bp	67:10	67:11	4	4.41
										May 15	ci	66:17	66:18	4	4.41
										Aug 15	ci	65:22	65:22	4	4.44
										Aug 15	bp	65:29	65:30	4	4.40
										Nov 15	ci	65:03	65:04	4	4.42
										Nov 15	bp	65:05	65:06	4	4.41
										Feb 16	ci	64:00	64:01	4	4.47
										Feb 16	bp	64:11	64:11	4	4.42
										May 16	ci	63:07	63:08	3	4.49
										May 16	bp	63:17	63:18	4	4.46
										Aug 16	ci	62:17	62:17	4	4.49
										Aug 16	ci	61:23	61:23	4	4.51
										Nov 16	bp	62:01	62:01	4	4.46
RATE	MATURITY MO/YR	BID	ASKED	CHG	ASK YLD	RATE	MATURITY MO/YR	BID	ASKED	CHG	ASK YLD				
Government Bonds & Notes															
1.875	Jan 06n	99:29	99:30	1	3.76	3.875	Feb 13n	97:10	97:11	4	4.31				
5.625	Feb 06n	100:03	100:04	...	3.82	3.625	May 13n	95:25	95:26	5	4.29				
9.375	Feb 06n	100:12	100:13	...	3.68	1.875	Jul 13n	99:27	99:28	9	1.89				
1.625	Feb 06n	99:22	99:23	1	4.16	4.250	Aug 13n	99:16	99:17	5	4.32				
1.500	Mar 06n	99:13	99:14	...	4.31	12,000	Aug 13	118:20	118:21	-1	4.26				
2.250	Apr 06n	99:12	99:13	...	4.33	4.250	Nov 13n	99:14	99:15	4	4.33				
6.500	Feb 10n	108:04	108:05	2	4.29	2,000	Jan 14n	100:19	100:20	9	1.92				
4,000	Mar 10n	98:29	98:30	3	4.28	4,000	Feb 14n	97:22	97:23	4	4.34				
0.875	Apr 10n	95:30	95:31	5	1.87	4,750	May 14n	102:27	102:28	5	4.33				
4,000	Apr 10n	98:28	98:29	3	4.28	13,250	May 14	127:12	127:13	1	4.30				
3.875	May 10n	98:13	98:14	3	4.27	2,000	Jul 14n	100:20	100:21	10	1.91				
3.625	Jun 10n	97:11	97:12	2	4.28	7,625	Feb 25	138:18	138:19	10	4.57				
3.875	Jul 10n	98:11	98:12	3	4.28	6,875	Aug 25	129:16	129:17	9	4.57				
5.750	Aug 10n	106:03	106:04	2	4.26	6,000	Feb 26	118:19	118:20	9	4.57				
4.125	Aug 10n	99:10	99:11	2	4.28	6,750	Aug 26	128:25	128:26	9	4.57				
3.675	Sep 10n	98:09	98:10	3	4.28	6,500	Nov 26	125:22	125:23	10	4.57				
4.250	Oct 10n	99:26	99:27	3	4.29	6,625	Feb 27	127:18	127:19	9	4.57				
4.500	Nov 10n	100:28	100:29	3	4.29	6,375	Aug 27	124:19	124:20	11	4.57				
4.375	Dec 10n	100:12	100:13	3	4.28	6,125	Nov 27	121:11	121:12	10	4.57				
4.250	Jan 11n	99:26	99:27	2	4.27	5,500	Apr 28n	130:00	130:00	15	1.95				
3.500	Jan 11n	107:25	107:26	6	1.85	5,250	Aug 28n	113:03	113:04	9	4.56				
5.000	Feb 11n	103:09	103:10	3	4.27	5,250	Nov 28n	109:24	109:25	10	4.55				
13.875	May 11n	103:02	103:03	...	9.81	3,875	Feb 29n	109:24	109:25	10	4.56				
5.000	Aug 11n	103:17	103:18	4	4.27	6,125	Apr 29n	135:22	135:23	10	1.95				
14.000	Nov 11n	107:25	107:26	...	4.19	6,250	Aug 29n	122:16	122:17	10	4.55				
3.375	Jan 12n	108:16	108:17	8	1.86	5,375	May 30n	124:28	124:29	10	4.55				
4.875	Feb 12n	103:00	103:01	3	4.30	3,375	Feb 31n	112:28	112:29	9	4.51				
3.000	Jul 12n	106:26	106:27	8	1.87	3,375	Apr 32n	131:01	131:02	15	1.87				
4.375	Aug 12n	100:11	100:12	3	4.31	U.S. Treasury Strips									
4.000	Nov 12n	98:06	98:07	4	4.30	MATURITY	TYPE	BID	ASKED	CHG	ASK YLD				
10.375	Nov 12	110:13	110:14	...	4.34	Feb 06	ci	99:23	99:23	...	3.53				

Source: BKM (2007)

Corporate Bond Listings

Corporate Bonds

Tuesday, January 24, 2006

Forty most active fixed-coupon corporate bonds

COMPANY (TICKER)	COUPON	MATURITY	LAST PRICE	LAST YIELD	*EST SPREAD	UST†	EST \$ VOL (000's)
Goldman Sachs Group Inc (GS)	5.350	Jan 15, 2016	100.052	5.343	95	10	223,906
HSBC Finance Corp (HSBC)	5.500	Jan 19, 2016	100.509	5.433	104	10	163,669
American General Finance Corp (AIG)	5.400	Dec 01, 2015	100.137	5.381	100	10	129,667
Verizon Global Funding Corp (VZ)	5.850	Sep 15, 2035	96.025	6.142	157	30	126,135
COX Communications Inc (COXENT)	7.125	Oct 01, 2012	106.942	5.852	146	5	111,982
Citigroup Inc (C)	5.300	Jan 07, 2016	101.104	5.155	78	10	108,173
HSBC Finance Corp (HSBC)	4.625	Sep 15, 2010	98.084	5.093	78	5	99,495

Volume represents total volume for each issue; price/yield data are for trades of \$1 million and greater. *Estimated spreads, in basis points (100 basis points is one percentage point), over the 2, 3, 5, 10 or 30-year hot run Treasury note/bond. 2-year: 4.375 12/07; 3-year: 4.375 11/08; 5-year: 4.250 01/11; 10-year: 4.500 11/15; 30-year: 5.375 02/31. †Comparable U.S. Treasury issue.

Source: MarketAxess Corporate BondTicker

Source: BKM (2007)

Provisions of Bonds

- Secured or unsecured
- Call provision
- Convertible provision
- Put provision (puttable bonds)
- Floating rate bonds
- Sinking funds

Principal and Interest Payments for Treasury Inflation Protected Security

Time	Inflation in Year Just Ended	Par Value	Coupon Payment	+	Principal Repayment	=	Total Payment
0		\$1,000.00					
1	2%	1,020.00	\$40.80		\$ 0		\$ 40.80
2	3	1,050.60	42.02		0		42.02
3	1	1,061.11	42.44		1,061.11		1,103.55

Source: BKM (2007)

Bond Pricing

$$P_B = \sum_{t=1}^T \frac{C_t}{(1+r)^t} + \frac{ParValue_T}{(1+r)^T}$$

P_B = Price of the bond

C_t = interest or coupon payments

T = number of periods to maturity

r = semi-annual discount rate or the semi-annual yield to maturity

Price: 10-yr, 8% Coupon, Face = \$1,000

$$P = 40 \sum_{t=1}^{20} \frac{1}{(1.03)^t} + \frac{1000}{(1.03)^{20}}$$

$$P = \$1,148.77$$

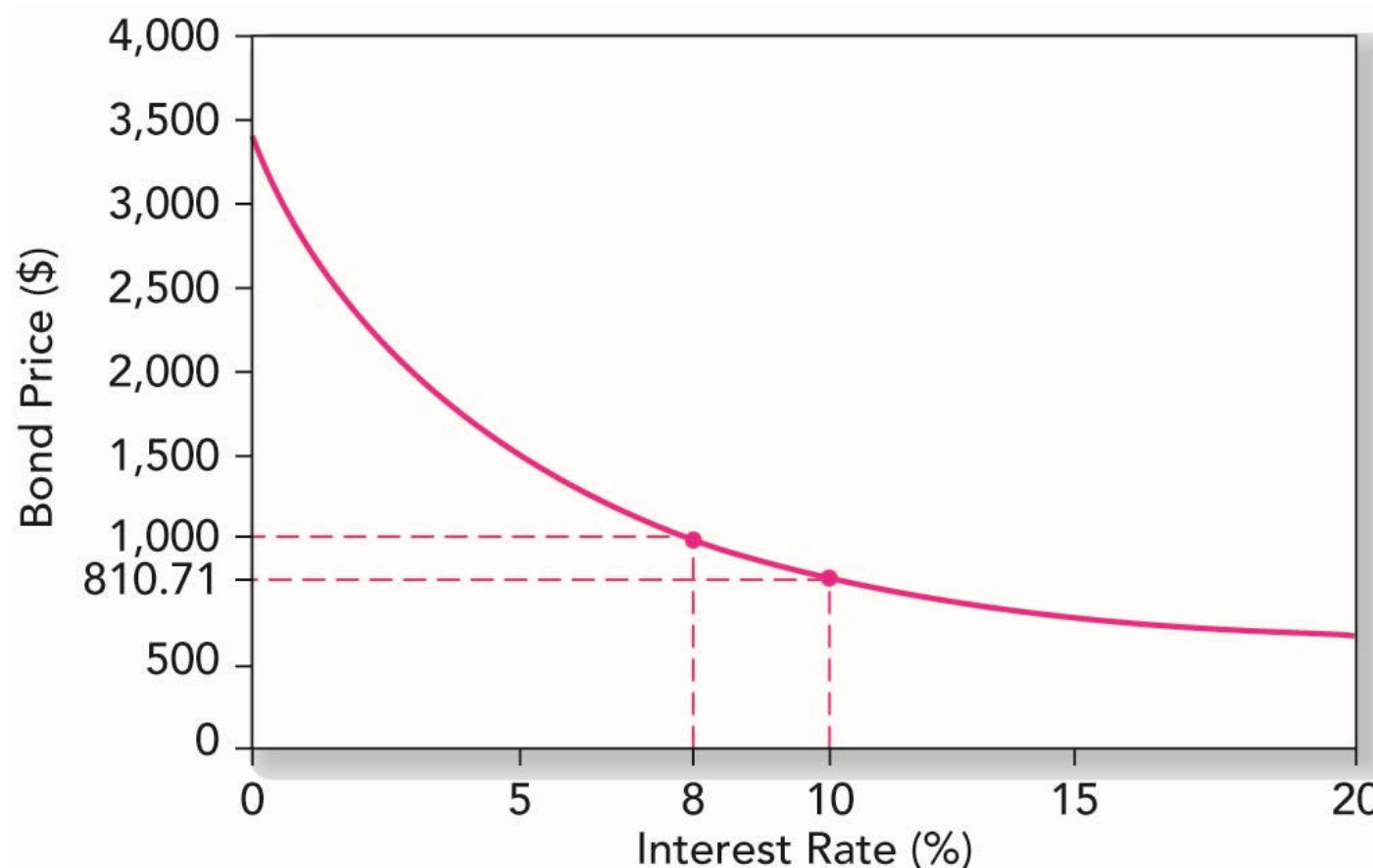
$$\begin{aligned} C_t &= 40 \text{ (SA)} \\ P &= 1000 \\ T &= 20 \text{ periods} \\ r &= 3\% \text{ (SA)} \end{aligned}$$

Bond Prices and Yields

Prices and Yields (required rates of return) have an inverse relationship

- When yields get very high the value of the bond will be very low.
- When yields approach zero, the value of the bond approaches the sum of the cash flows.

The Inverse Relationship Between Bond Prices and Yields



Source: BKM (2007)

Bond Prices at Different Interest Rates (8% Coupon Bond, Coupons Paid Semiannually)

Time to Maturity	Bond Price at Given Market Interest Rate				
	4%	6%	8%	10%	12%
1 year	1,038.83	1,029.13	1,000.00	981.41	963.33
10 years	1,327.03	1,148.77	1,000.00	875.35	770.60
20 years	1,547.11	1,231.15	1,000.00	828.41	699.07
30 years	1,695.22	1,276.76	1,000.00	810.71	676.77

Source: BKM (2007)

Yield to Maturity

- Interest rate that makes the present value of the bond's payments equal to its price.

Solve the bond formula for r

$$P_B = \sum_{t=1}^T \frac{C_t}{(1+r)^t} + \frac{ParValue_T}{(1+r)^T}$$

Yield to Maturity Example

$$950 = \sum_{t=1}^{20} \frac{35}{(1+r)^t} + \frac{1000}{(1+r)^T}$$

10 yr Maturity Coupon Rate = 7%

Price = \$950

Solve for r = semiannual rate

r = 3.8635%



Yield Measures

Bond Equivalent Yield

$$7.72\% = 3.86\% \times 2$$

Effective Annual Yield

$$(1.0386)^2 - 1 = 7.88\%$$

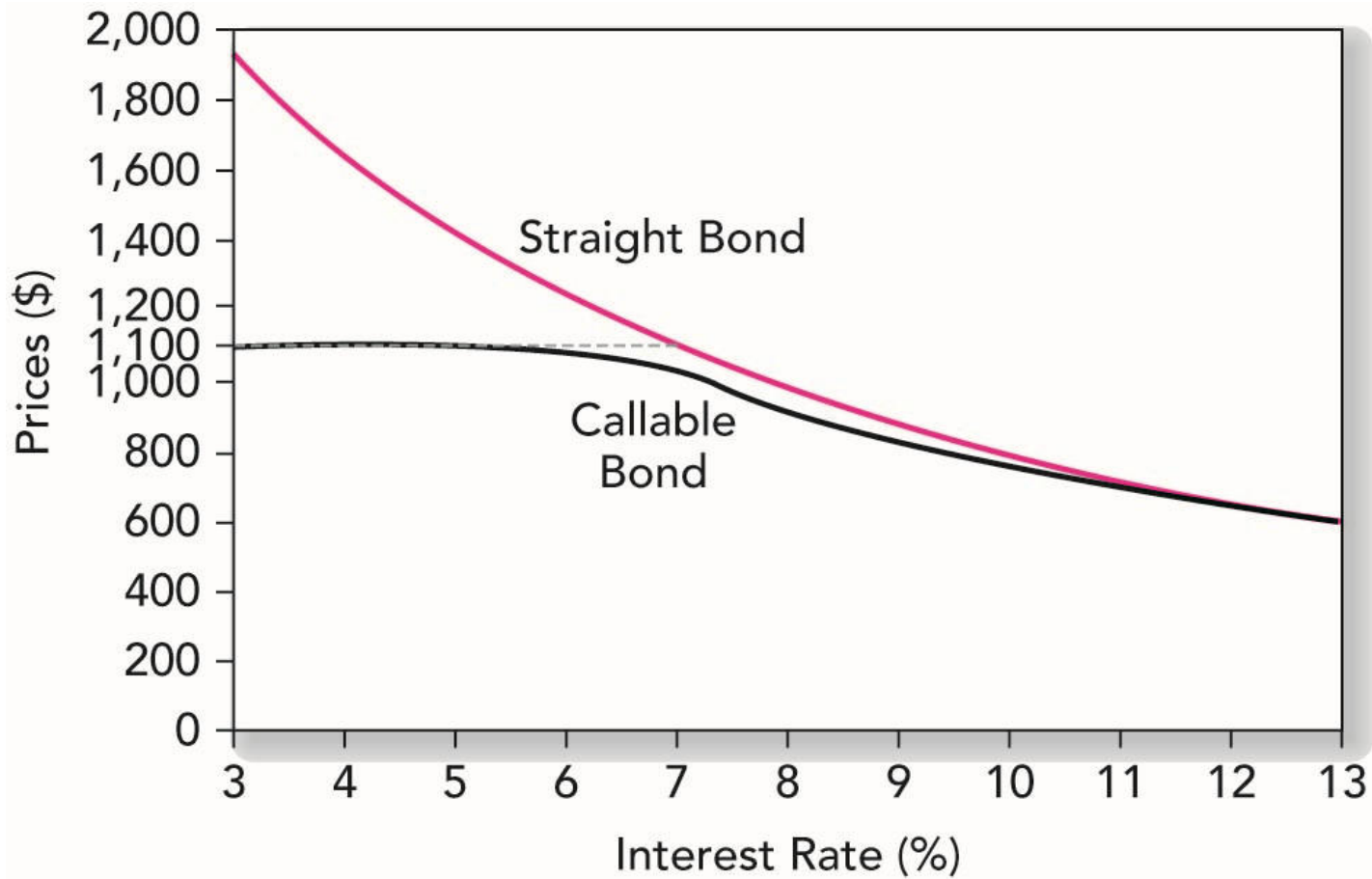
Current Yield

Annual Interest / Market Price

$$\$70 / \$950 = 7.37\%$$

Yield to Call

Bond Prices: Callable and Straight Debt



Source: BKM (2007)

Yield to Call

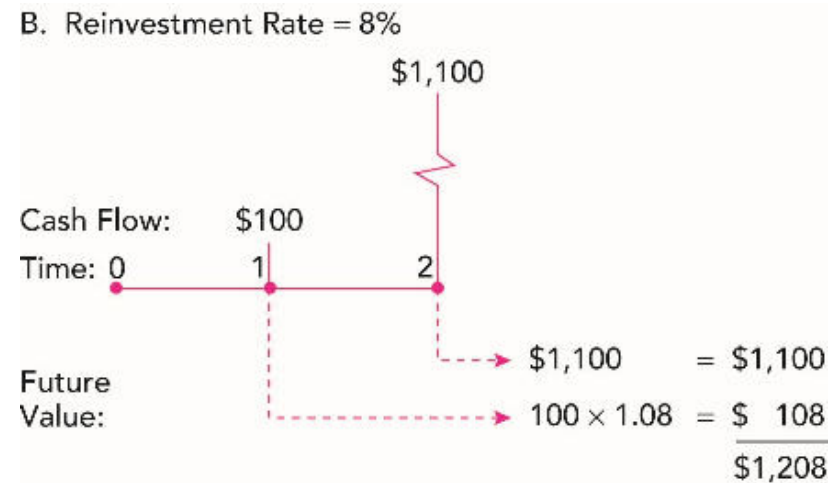
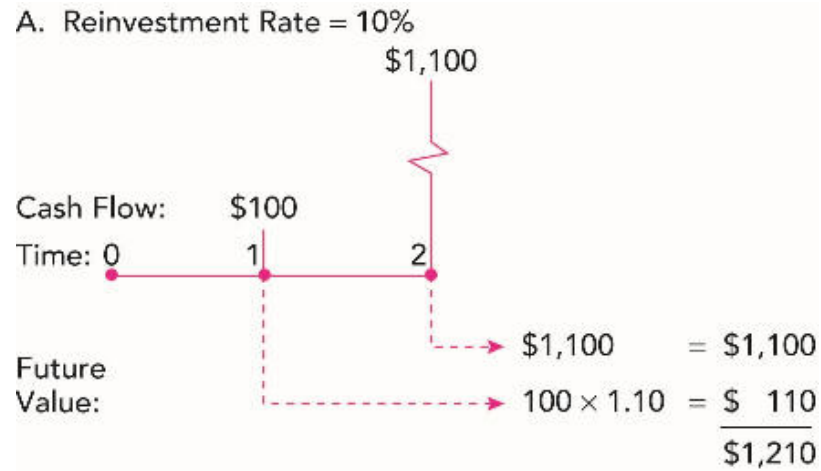
	Yield to Call	Yield to Maturity
Coupon payment	\$40	\$40
Number of semiannual periods	20 periods	60 periods
Final payment	\$1,100	\$1,000
Price	\$1,150	\$1,150

Source: BKM (2007)

Realized Yield versus YTM

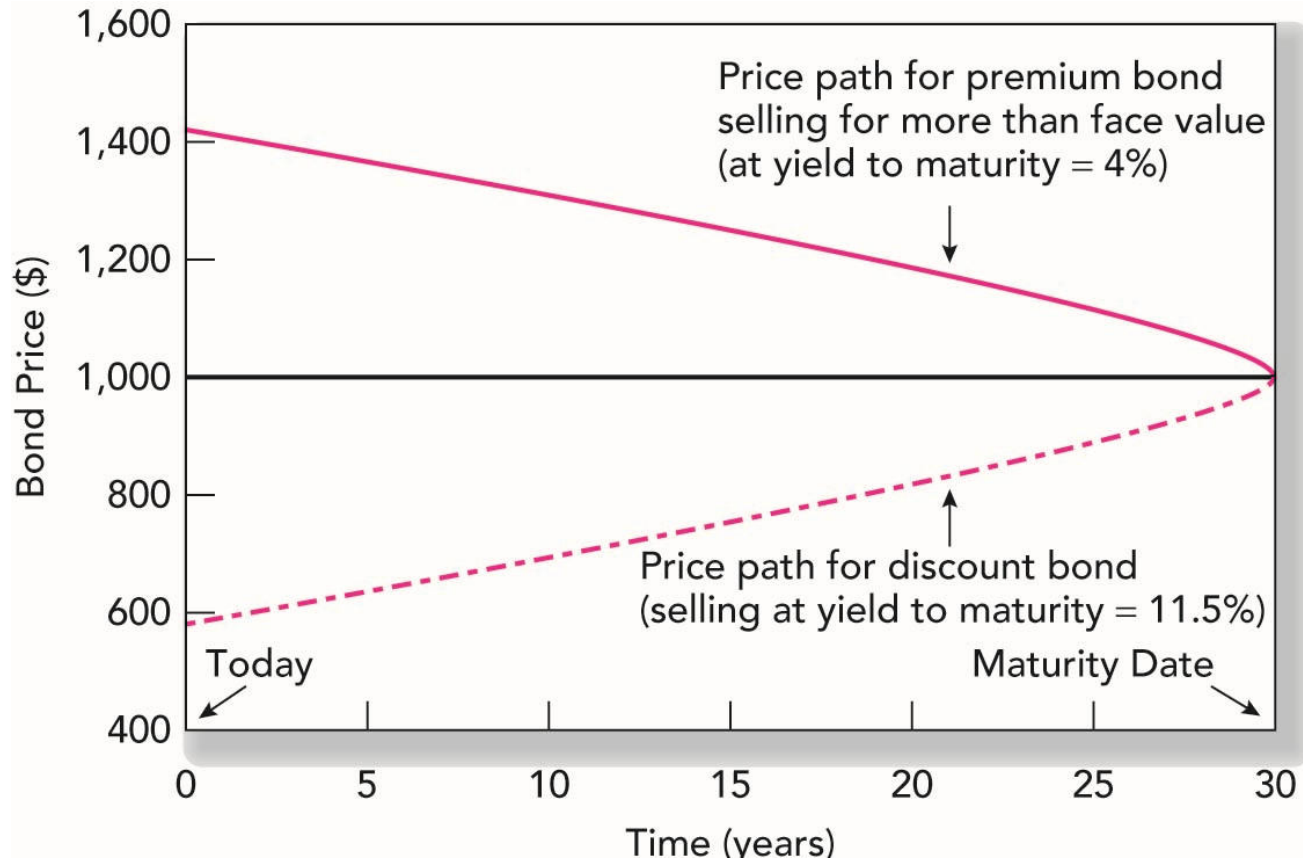
- Reinvestment Assumptions
- Holding Period Return
 - Changes in rates affects returns
 - Reinvestment of coupon payments
 - Change in price of the bond

Growth of Invested Funds



Source: BKM (2007)

Prices over Time of 30-Year Maturity, 6.5% Coupon Bonds



Source: BKM (2007)



Holding-Period Return: Single Period

$$\text{HPR} = [I + (-P_0 + P_1)] / P_0$$

where

I = interest payment

P_1 = price in one period

P_0 = purchase price

Holding-Period Example

$$CR = 8\%$$

$$YTM = 8\%$$

$$N=10 \text{ years}$$

Semiannual Compounding

$$P_0 = \$1000$$

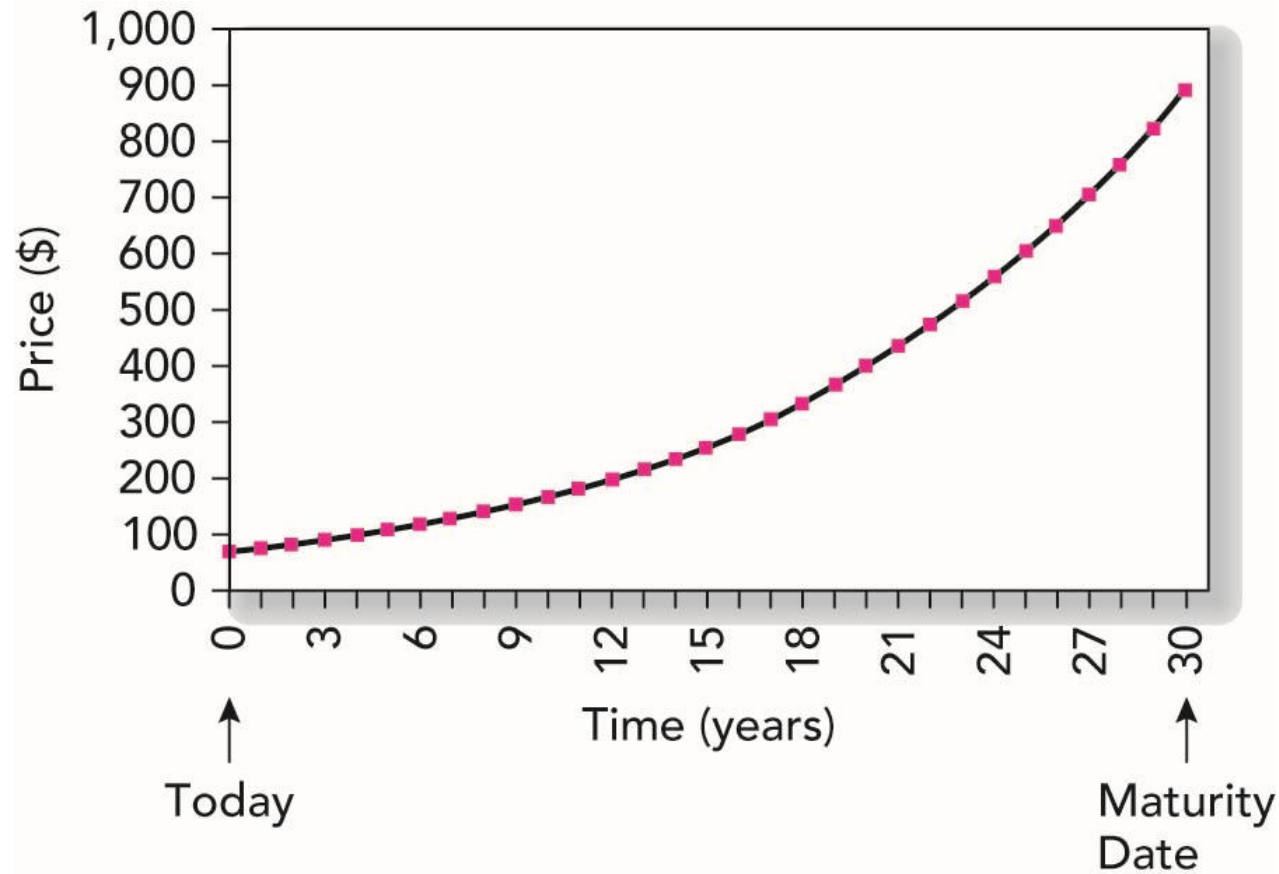
In six months the rate falls to 7%

$$P_1 = \$1068.55$$

$$HPR = [40 + (1068.55 - 1000)] / 1000$$

$$HPR = 10.85\% \text{ (semiannual)}$$

The Price of a 30-Year Zero-Coupon Bond over Time at a Yield to Maturity of 10%



Source: BKM (2007)

Default Risk and Ratings

- Rating companies
 - Moody's Investor Service
 - Standard & Poor's
 - Fitch
- Rating Categories
 - Investment grade
 - Speculative grade

Definitions of Each Bond Rating Class

Bond Ratings								
	Very High Quality		High Quality		Speculative		Very Poor	
Standard & Poor's	AAA	AA	A	BBB	BB	B	CCC	D
Moody's	Aaa	Aa	A	Baa	Ba	B	Caa	C
<p>At times both Moody's and Standard & Poor's have used adjustments to these ratings: S&P uses plus and minus signs: A + is the strongest A rating and A - the weakest. Moody's uses a 1, 2, or 3 designation, with 1 indicating the strongest.</p>								
Moody's	S&P							
Aaa	AAA	Debt rated Aaa and AAA has the highest rating. Capacity to pay interest and principal is extremely strong.						
Aa	AA	Debt rated Aa and AA has a very strong capacity to pay interest and repay principal. Together with the highest rating, this group comprises the high-grade bond class.						
A	A	Debt rated A has a strong capacity to pay interest and repay principal, although it is somewhat more susceptible to the adverse effects of changes in circumstances and economic conditions than debt in higher-rated categories.						
Baa	BBB	Debt rated Baa and BBB is regarded as having an adequate capacity to pay interest and repay principal. Whereas it normally exhibits adequate protection parameters, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity to pay interest and repay principal for debt in this category than in higher-rated categories. These bonds are medium-grade obligations.						
Ba	BB	Debt rated in these categories is regarded, on balance, as predominantly speculative with respect to capacity to pay interest and repay principal in accordance with the terms of the obligation. BB and Ba indicate the lowest degree of speculation, and CC and Ca the highest degree of speculation. Although such debt will likely have some quality and protective characteristics, these are outweighed by large uncertainties or major risk exposures to adverse conditions. Some issues may be in default.						
B	B							
Caa	CCC							
Ca	CC	This rating is reserved for income bonds on which no interest is being paid.						
C	C	Debt rated D is in default, and payment of interest and/or repayment of principal is in arrears.						
D	D							

Source: BKM (2007)



Factors Used by Rating Companies

- Coverage ratios:
EBITDA*/interest payments: a measurement of the number of times a company can make its interest payments with its earnings before interest and taxes.
*Earning before interest tax depreciation and amortization
- Leverage ratios:
 - 1) Debt/Equity
Measures how much of the company is financed by its debtholders compared with its owners Liquidity ratios
 - 2) Interest Coverage = $\text{Operating Income} / \text{Interest Expense}$
Measures a company's ability to meet its interest obligations with income earned from the firm's primary source of business.
- Profitability ratios
 - 1) Gross Margin = $\text{Gross Profit} / \text{Sales}$
 - 2) Operating Margin = $\text{Operating Income or Loss} / \text{Sales}$
 - 3) Net Margin = $\text{Net Income or Loss} / \text{Sales}$
 - 4) Return on Assets = $(\text{Net Income} + \text{Aftertax Interest Expense}) / \text{Average Total Assets}$
 - 5) Return on Equity = $\text{Net Income} / \text{Average Shareholders' Equity}$
- Cash flow to debt: $\text{Operating Cash Flow} / \text{Total Debt}$

Financial Ratios and Default Risk by Rating Class, Long-Term Debt

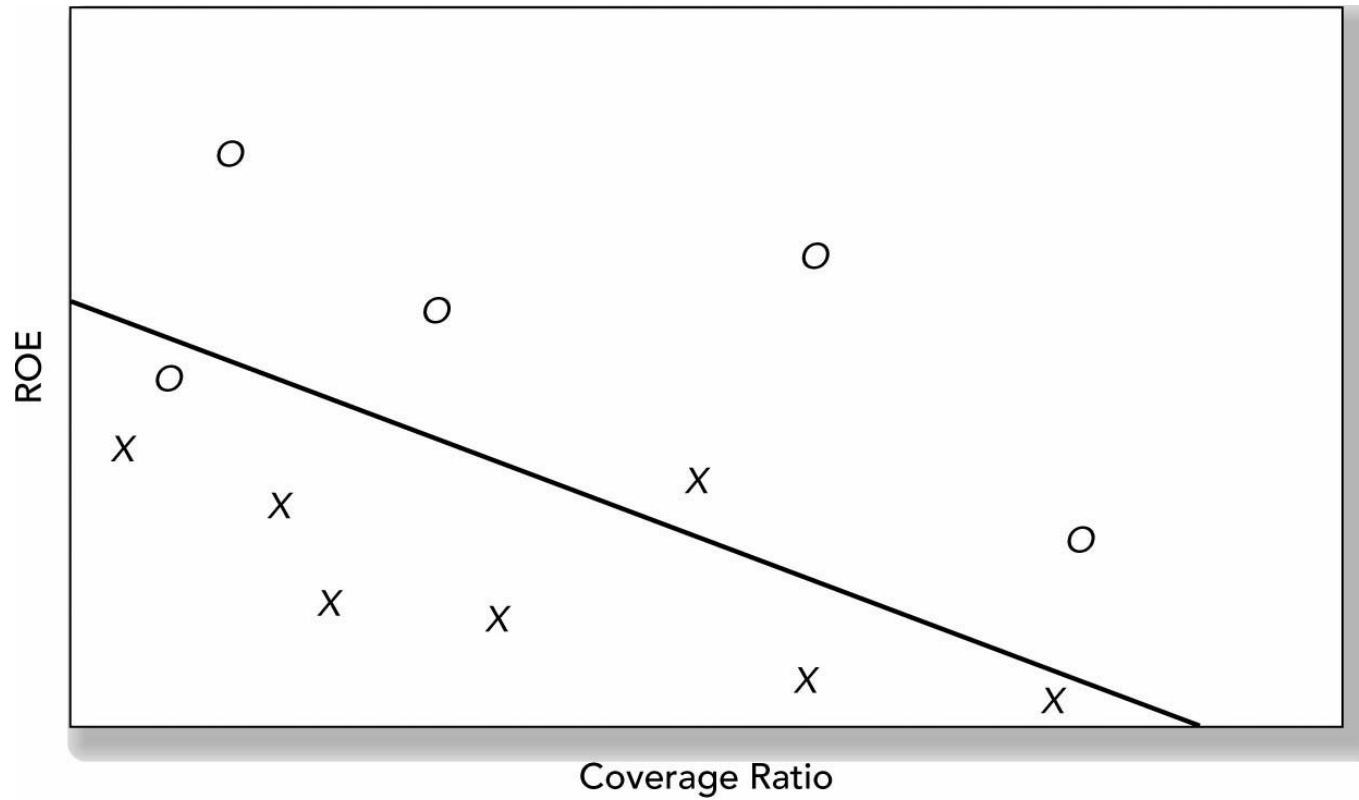
	Three-year (2002 to 2004) medians						
	AAA	AA	A	BBB	BB	B	CCC
EBIT interest coverage multiple	23.8	19.5	8.0	4.7	2.5	1.2	0.4
EBITDA interest coverage multiple	25.5	24.6	10.2	6.5	3.5	1.9	0.9
Funds from operations/total debt (%)	203.3	79.9	48.0	35.9	22.4	11.5	5.0
Free operating cash flow/total debt (%)	127.6	44.5	25.0	17.3	8.3	2.8	(2.1)
Total debt/EBITDA multiple	0.4	0.9	1.6	2.2	3.5	5.3	7.9
Return on capital (%)	27.6	27.0	17.5	13.4	11.3	8.7	3.2
Total debt/total debt + equity (%)	12.4	28.3	37.5	42.5	53.7	75.9	113.5
Historical default rate (%)	0.5	1.3	2.3	6.6	19.5	35.8	54.4

Note: EBITDA is earnings before interest, taxes, depreciation, and amortization

Source: *Corporate Rating Criteria*, Standard & Poor's, 2006. Historical default rates from "Static Pools Cumulative Average Default Rates (%)," Standard & Poor's. Reproduced by permission of Standard & Poor's, a division of The McGraw-Hill Companies, Inc.

Source: BKM (2007)

Discriminant Analysis



Source: BKM (2007)

Protection Against Default

- Sinking funds
- Subordination of future debt
- Dividend restrictions
- Collateral

Callable Bond Issue by Mobil

**& Mobil Corp. debenture 8s, due 2032:
Rating — Aa2**

AUTH—\$250,000,000.
 OUTSTG—Dec. 31, 1993, \$250,000,000.
 DATED—Oct. 30, 1991.
 INTEREST—F&A 12.
 TRUSTEE—Chemical Bank.
 DENOMINATION—Fully registered, \$1,000 and integral multiples thereof. Transferable and exchangeable without service charge.
 CALLABLE—As a whole or in part, at any time, on or after Aug. 12, 2002, at the option of Co. on at least 30 but not more than the 60 days' notice to each Aug. 11 as follows:

2003.....105.007	2004.....104.756	2005.....104.506
2006.....104.256	2007.....104.005	2008.....103.755
2009.....103.505	2010.....103.254	2011.....103.004
2012.....102.754	2013.....102.503	2014.....102.253
2015.....102.003	2016.....101.752	2017.....101.502
2018.....101.252	2019.....101.001	2020.....100.751
2021.....100.501	2022.....100.250	

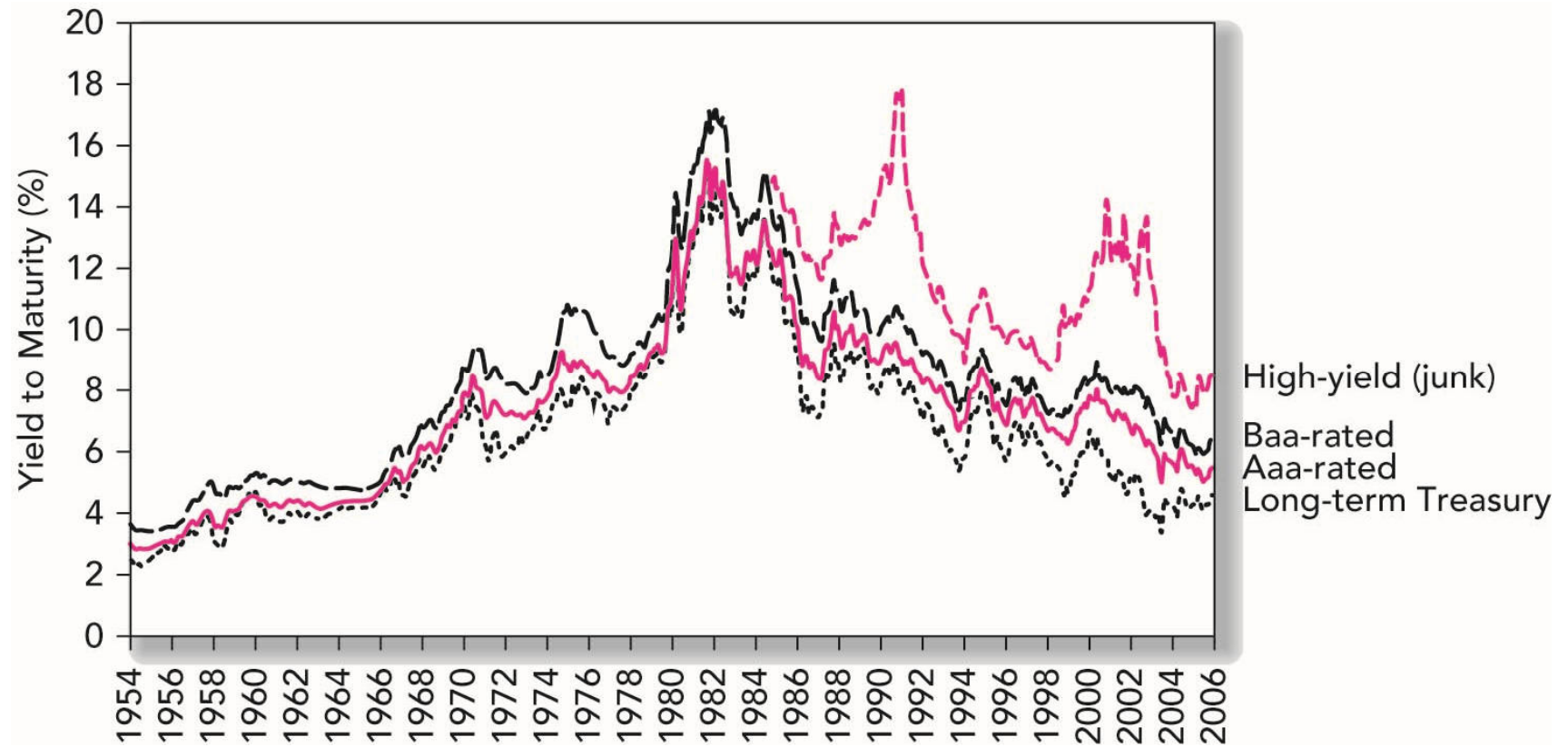
and thereafter at 100 plus accrued interest.
 SECURITY—Not secured. Ranks equally with all other unsecured and unsubordinated indebtedness of Co. Co. nor any Affiliate will not incur any indebtedness; provided that Co. will not create as security for any indebtedness for borrowed money, any mortgage, pledge, security interest or lien on any stock or indebtedness is directly owned by Co. without effectively providing that the debt securities shall be secured equally and ratably with such indebtedness, so long as such indebtedness shall be so secured.
 INDENTURE. MODIFICATION—Indenture may be modified, except as provided with, consent of 66 2/3% of debts. outstg.
 RIGHTS ON DEFAULT—Trustee, or 25% of debts. outstg., may declare principal due and payable (30 days' grace for payment of interest).
 LISTED—On New York Stock Exchange.
 PURPOSE—Proceeds used for general corporate purposes.
 OFFERED—(\$250,000,000) at 99.51 plus accrued interest (proceeds to Co., 99.11) on Aug. 5, 1992 thru Merrill Lynch & Co., Donaldson, Lufkin & Jenerette Securities Corp., PaineWebber Inc., Prudential Securities Inc., Smith Barney, Harris Upham & Co. Inc. and associates.

Source: BKM (2007)

Default Risk and Yield

- Risk structure of interest rates
- Default premiums
 - Yields compared to ratings
 - Yield spreads over business cycles

Yields on Long-Term Bonds, 1954 – 2006



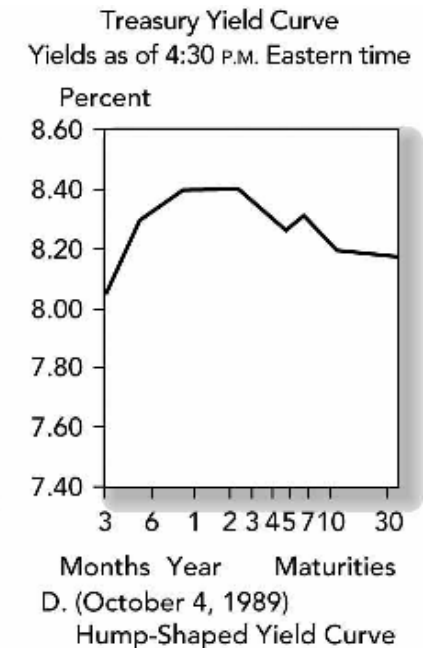
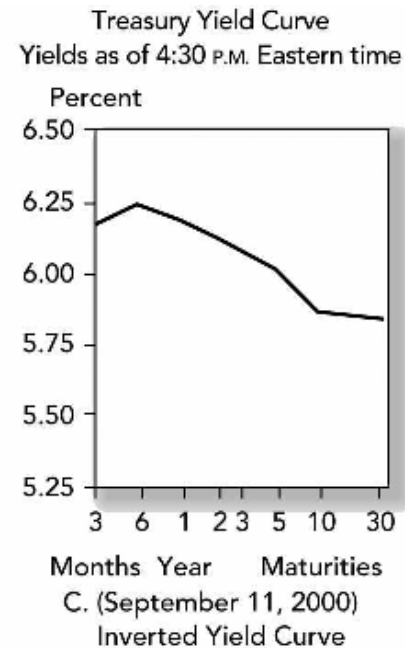
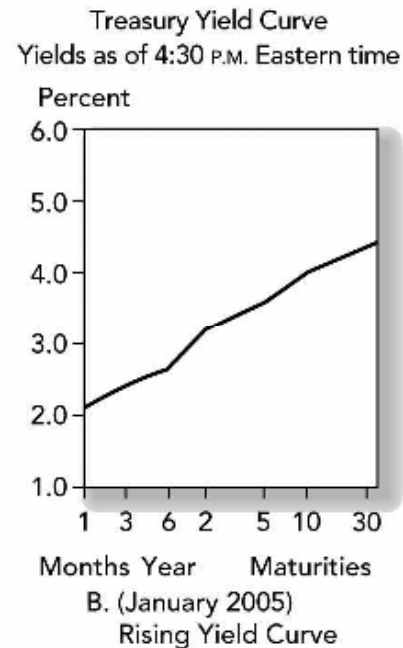
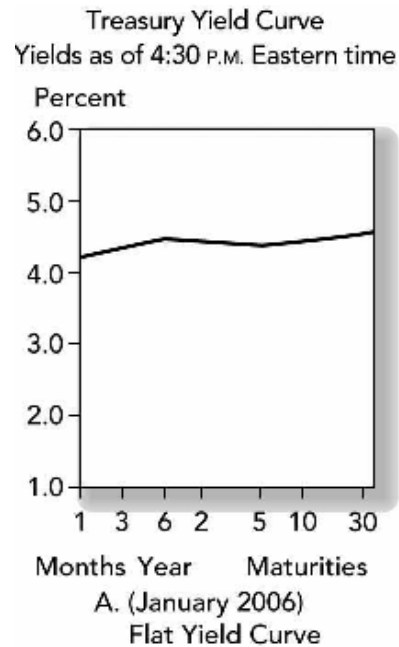
Source: BKM (2007)

8.2 The Term-Structure of Interest Rates

Overview of Term Structure

- The relationship between yield to maturity and maturity.
- Information on expected future short term rates can be implied from yield curve.
- The yield curve is a graph that displays the relationship between yield and maturity.
- Three major theories are proposed to explain the observed yield curve.

Treasury Yield Curves



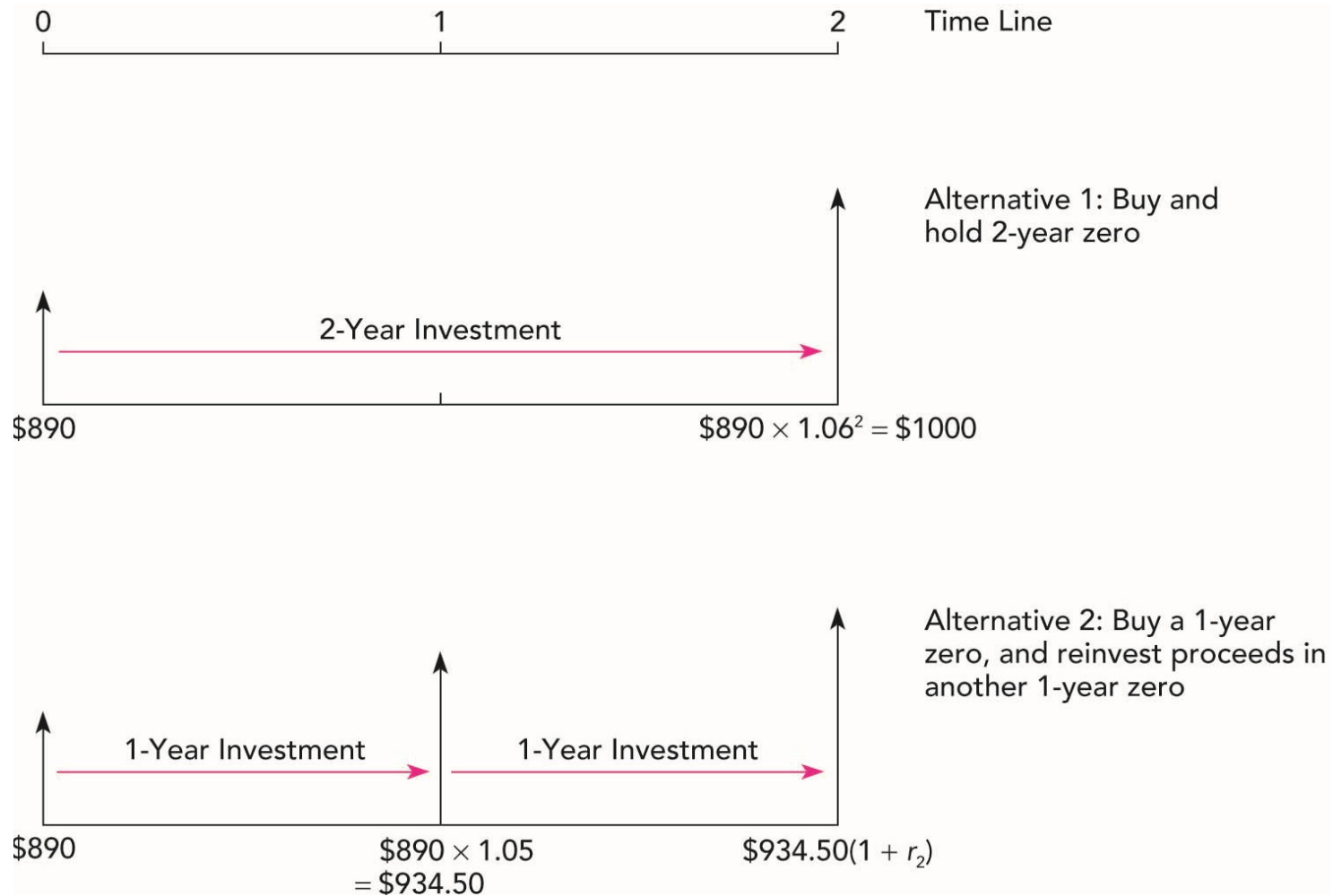
Source: BKM (2007)

Yield to Maturity and Prices and Prices on Zero-Coupon Bonds (\$1,000 Face Value)

Maturity (years)	Yield to Maturity (%)	Price
1	5%	$\$952.38 = \$1,000/1.05$
2	6	$\$890.00 = \$1,000/1.06^2$
3	7	$\$816.30 = \$1,000/1.07^3$
4	8	$\$735.03 = \$1,000/1.08^4$

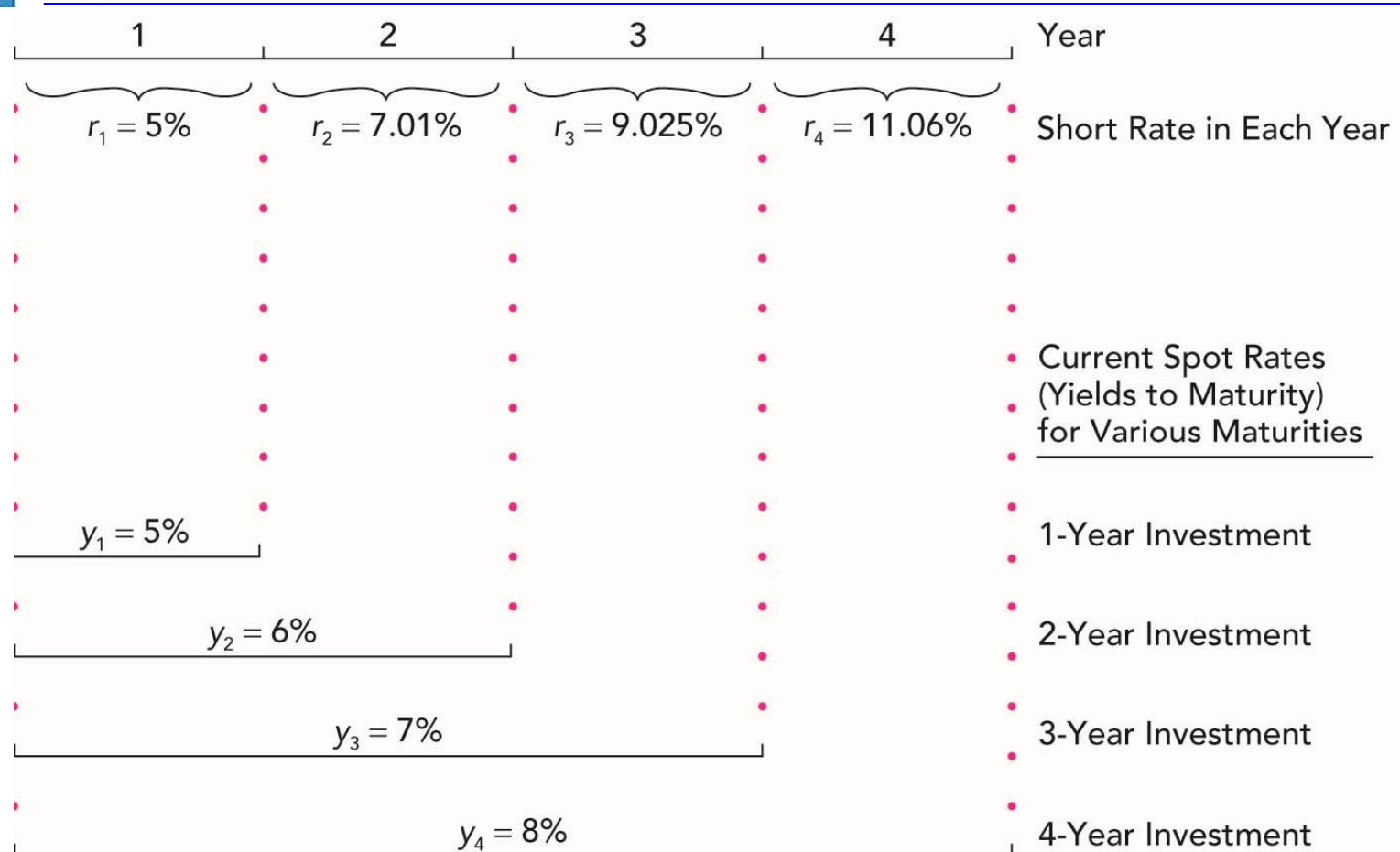
Source: BKM (2007)

Two 2-Year Investment Programs



Source: BKM (2007)

Short Rates versus Spot Rates



Source: BKM (2007)

Forward Rates from Observed Rates

$$(1 + f_n) = \frac{(1 + y_n)^n}{(1 + y_{n-1})^{n-1}}$$

f_n = one-year forward rate for period n

y_n = yield for a security with a maturity of n

$$(1 + y_n)^n = (1 + y_{n-1})^{n-1} (1 + f_n)$$

Example of Forward Rates

4 yr = 8.00% 3yr = 7.00% $f_n = ?$

$$(1.08)^4 = (1.07)^3 (1+f_n)$$

$$(1.3605) / (1.2250) = (1+f_n)$$

$$f_n = .1106 \text{ or } 11.06\%$$

Note: this is expected rate that was used in the prior example.

Downward Sloping Spot Yield Curve

Zero-Coupon Rates	Bond Maturity
12%	1
11.75%	2
11.25%	3
10.00%	4
9.25%	5

Forward Rates Downward Sloping Y C

1yr Forward Rates

$$1\text{yr} \quad [(1.1175)^2 / 1.12] - 1 \quad = \quad 0.115006$$

$$2\text{yrs} \quad [(1.1125)^3 / (1.1175)^2] - 1 \quad = \quad 0.102567$$

$$3\text{yrs} \quad [(1.1)^4 / (1.1125)^3] - 1 \quad = \quad 0.063336$$

$$4\text{yrs} \quad [(1.0925)^5 / (1.1)^4] - 1 \quad = \quad 0.063008$$

Theories of Term Structure

- Expectations
- Liquidity Preference
 - Upward bias over expectations

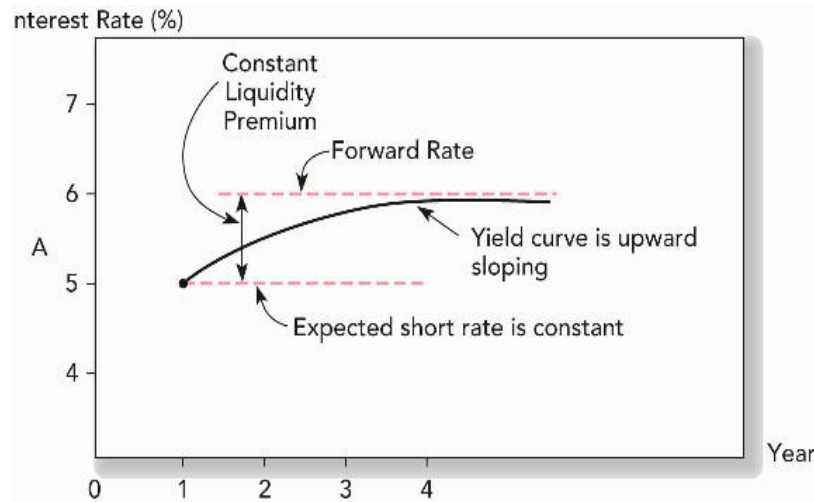
Expectations Theory

- Observed long-term rate is a function of today's short-term rate and expected future short-term rates.
- Long-term and short-term securities are perfect substitutes.
- Forward rates that are calculated from the yield on long-term securities are market consensus expected future short-term rates.

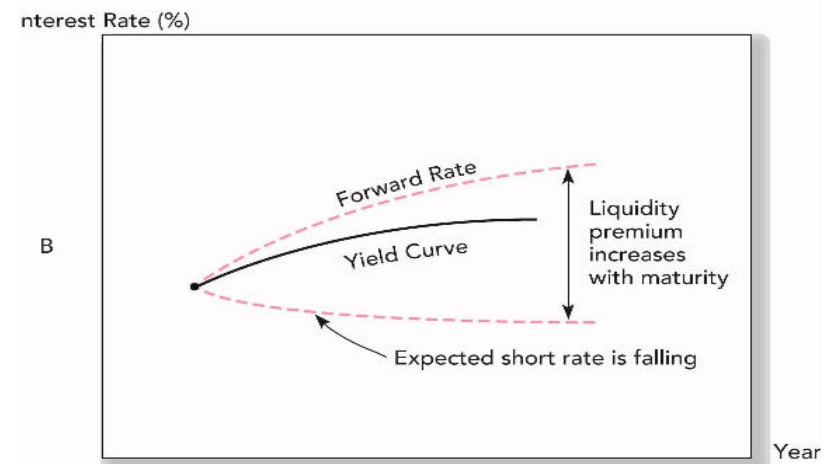
Liquidity Premium Theory

- Long-term bonds are more risky.
- Investors will demand a premium for the risk associated with long-term bonds.
- The yield curve has an upward bias built into the long-term rates because of the risk premium.
- Forward rates contain a liquidity premium and are not equal to expected future short-term rates.

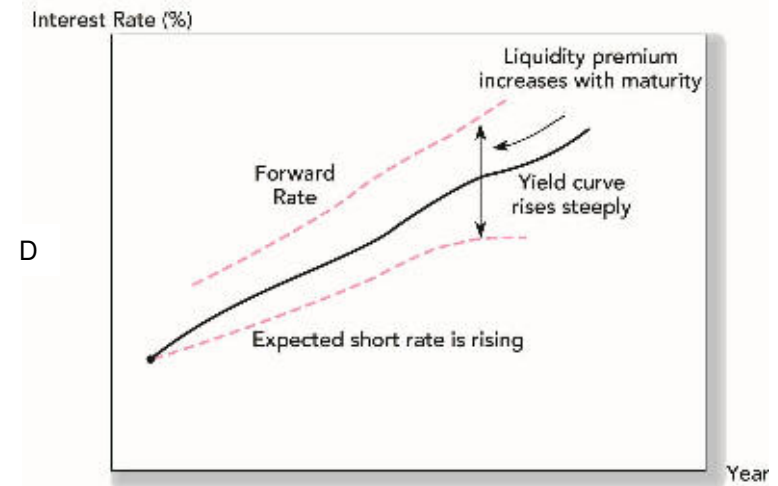
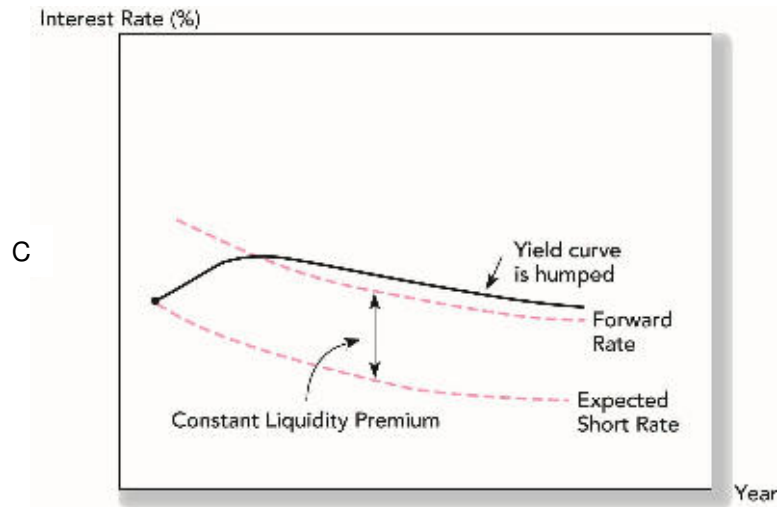
Yield Curves (with Liquidity Premiums)



Source: BKM (2007)

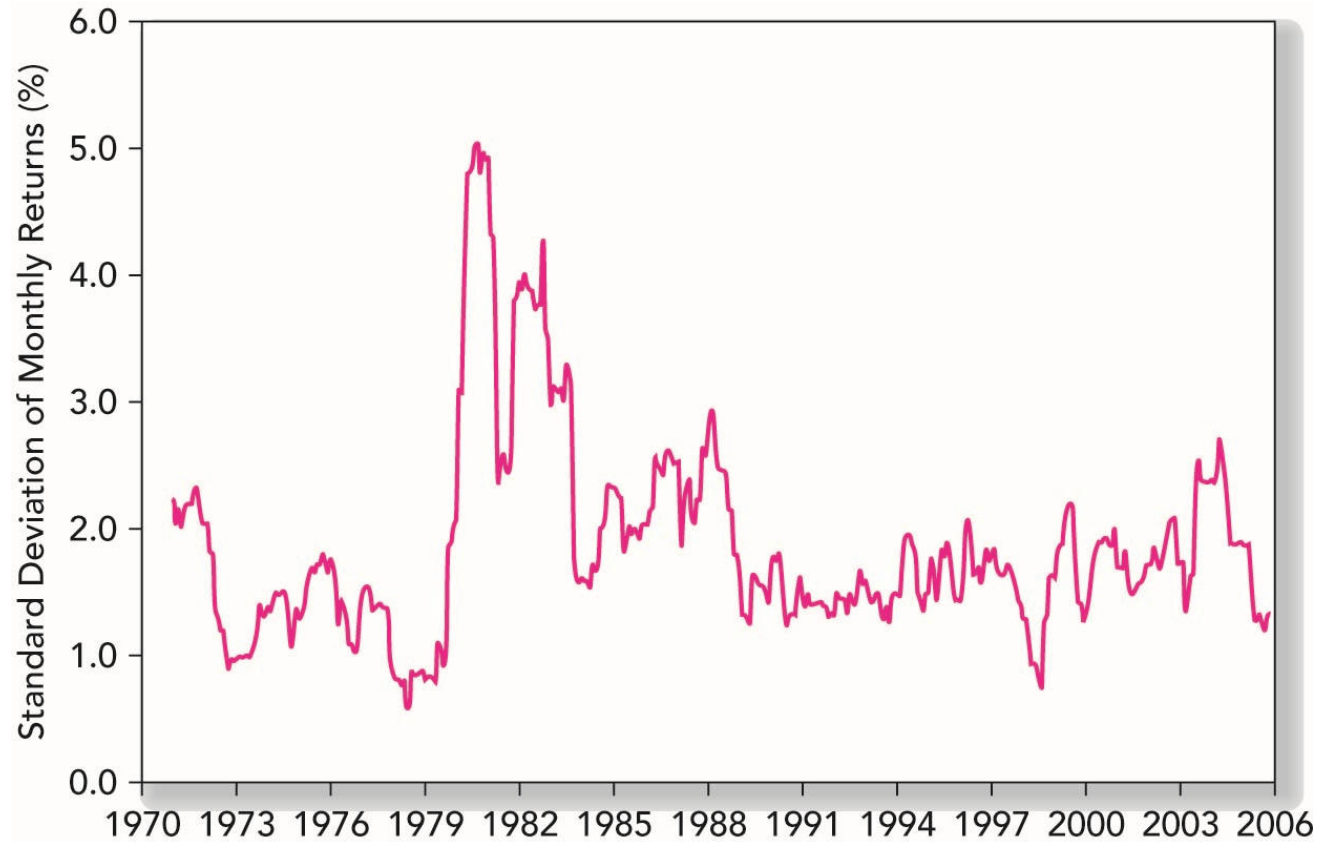


Yield Curves (Concluded)



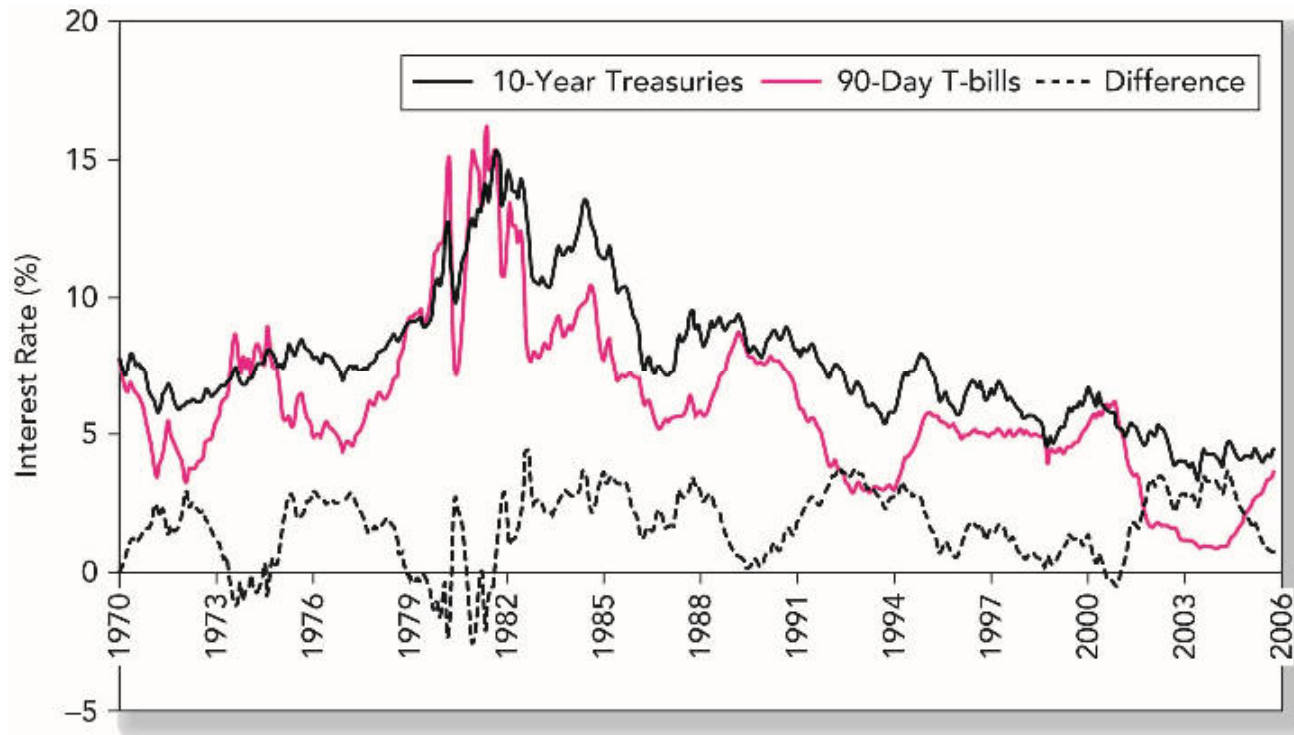
Source: BKM (2007)

Price Volatility of Long-Term Treasury Bonds



Source: BKM (2007)

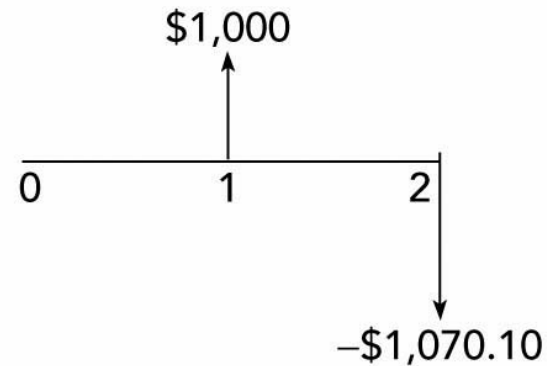
Yields on 10-Year versus 90-Day Treasury Securities: Term Spread



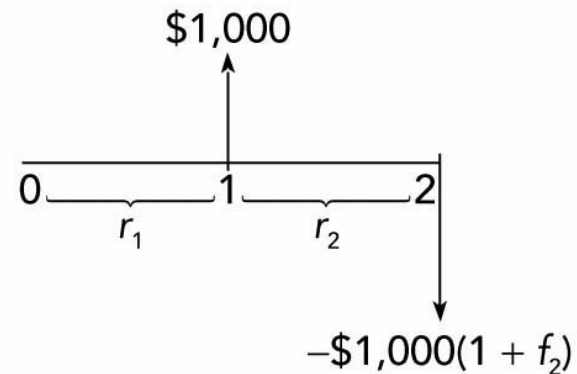
Source: BKM (2007)

Engineering a Synthetic Forward Loan

A. Forward Rate = 7.01%



B. For a General Forward Rate. The short rates in the two periods are r_1 (which is observable today) and r_2 (which is not). The rate that can be locked in for a one-period-ahead loan is f_2 .



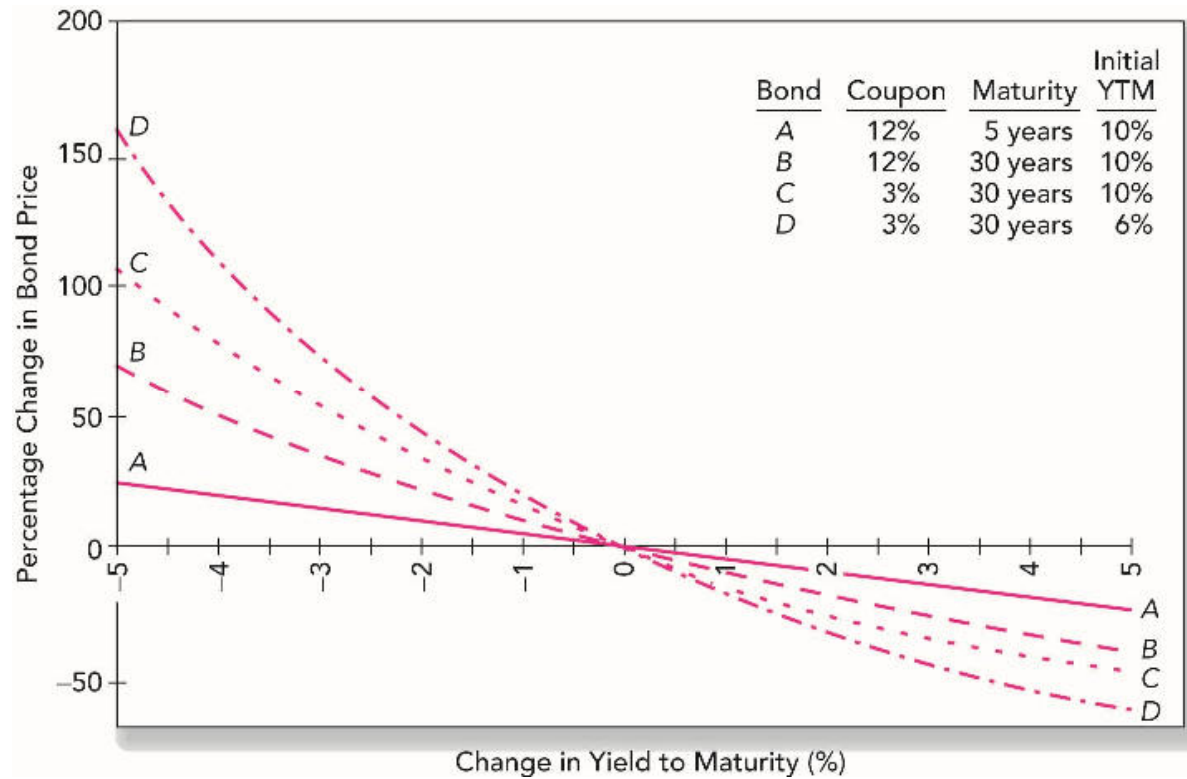
Source: BKM (2007)

8.3 Managing Bond Portfolios

Bond Pricing Relationships

- Inverse relationship between price and yield.
- An increase in a bond's yield to maturity results in a smaller price decline than the gain associated with a decrease in yield.
- Long-term bonds tend to be more price sensitive than short-term bonds.

Change in Bond Price as a Function of Change in Yield to Maturity



Source: BKM (2007)

Bond Pricing Relationships (cont'd)

- As maturity increases, price sensitivity increases at a decreasing rate.
- Price sensitivity is inversely related to a bond's coupon rate.
- Price sensitivity is inversely related to the yield to maturity at which the bond is selling.

Prices of an 8% Coupon Bond (Coupons Paid Semiannually)

Yield to Maturity (APR)	$T = 1$ Year	$T = 10$ Years	$T = 20$ Years
8%	1,000.00	1,000.00	1,000.00
9%	990.64	934.96	907.99
Fall in price (%)*	0.94%	6.50%	9.20%

*Equals value of bond at a 9% yield to maturity divided by value of bond at (the original) 8% yield, minus 1.

Source: BKM (2007)

Prices of Zero-Coupon Bond (Coupons Paid Semiannually)

Yield to Maturity (APR)	$T = 1$ Year	$T = 10$ Years	$T = 20$ Years
8%	924.56	456.39	208.29
9%	915.73	414.64	171.93
Fall in price (%)*	0.96%	9.15%	17.46%

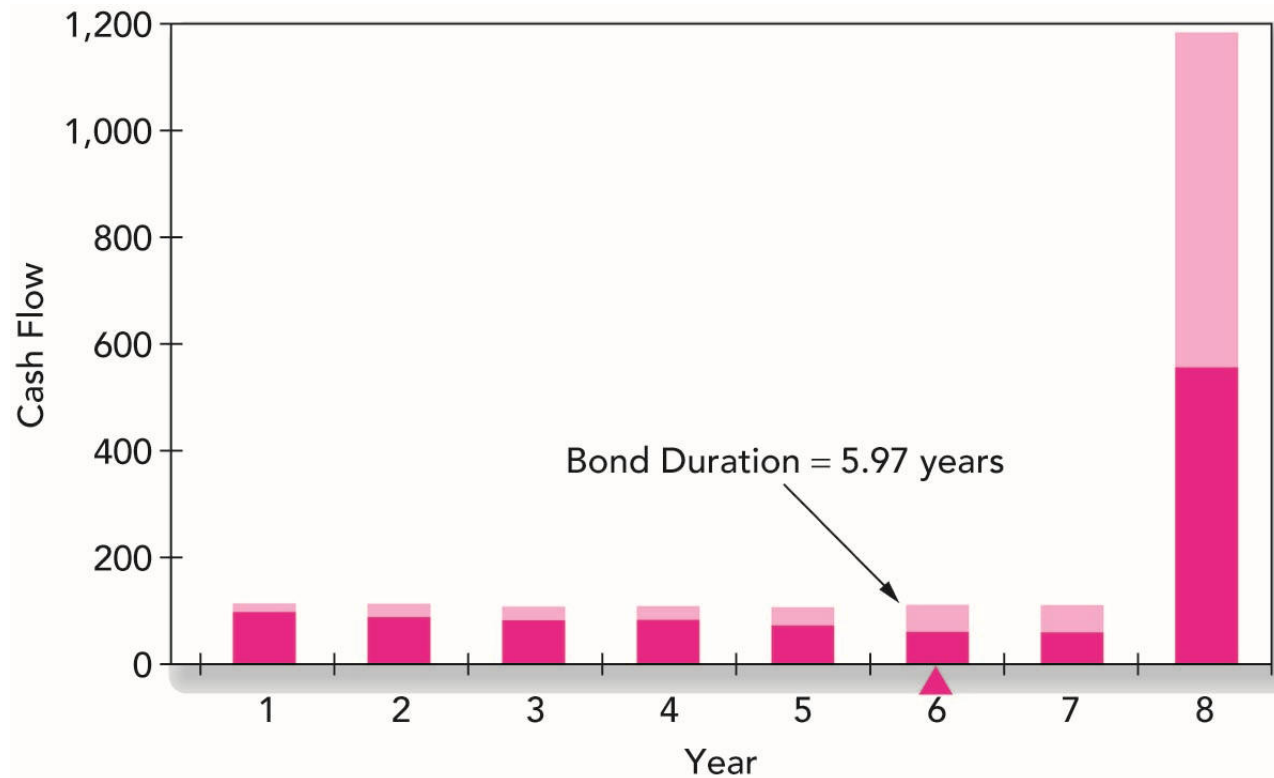
*Equals value of bond at a 9% yield to maturity divided by value of bond at (the original) 8% yield, minus 1.

Source: BKM (2007)

Duration

- A measure of the effective maturity of a bond.
- The weighted average of the times until each payment is received, with the weights proportional to the present value of the payment.
- Duration is shorter than maturity for all bonds except zero coupon bonds.
- Duration is equal to maturity for zero coupon bonds.

Cash Flows Paid by 9% Coupon, Annual Payment Bond with an 8-Year Maturity and 10% Yield to Maturity



Source: BKM (2007)

Duration: Calculation

$$w_t = \left[CF_t / (1 + y)^t \right] / \text{Price}$$

$$D = \sum_{t=1}^T t \times w_t$$

$CF_t = \text{Cash Flow for period } t$

Calculating the Duration of Two Bonds

	A	B	C	D	E	F	G
1			Time until		PV of CF		Column (C)
2			Payment		(Discount rate =		times
3		Period	(Years)	Cash Flow	5% per period)	Weight*	Column (F)
4	A. 8% coupon bond	1	0.5	40	38.095	0.0395	0.0197
5		2	1.0	40	36.281	0.0376	0.0376
6		3	1.5	40	34.554	0.0358	0.0537
7		4	2.0	1040	855.611	0.8871	1.7741
8	Sum:				964.540	1.0000	1.8852
9							
10	B. Zero-coupon	1	0.5	0	0.000	0.0000	0.0000
11		2	1.0	0	0.000	0.0000	0.0000
12		3	1.5	0	0.000	0.0000	0.0000
13		4	2.0	1000	822.702	1.0000	2.0000
14	Sum:				822.702	1.0000	2.0000
15							
16	Semiannual int rate:	0.05					
17							
18	*Weight = Present value of each payment (column E) divided by the bond price.						

Source: BKM (2007)

Duration/Price Relationship

Price change is proportional to duration and not to maturity.

$$\Delta P/P = -D \times [\Delta(1+y) / (1+y)]$$

D^* = *modified duration*

$$D^* = D / (1+y)$$

$$\Delta P/P = - D^* \times \Delta y$$

Rules for Duration

Rule 1 The duration of a zero-coupon bond equals its time to maturity.

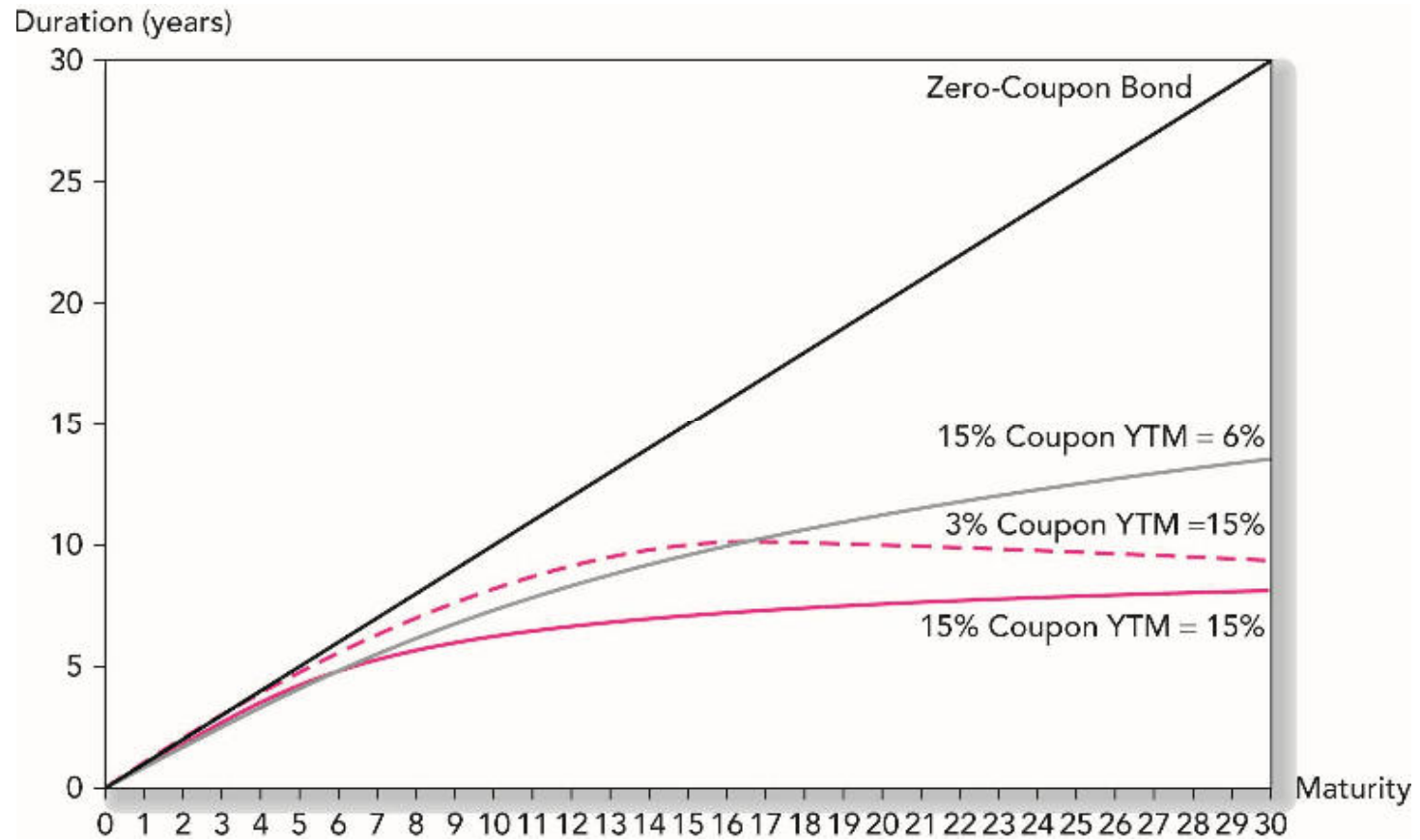
Rule 2 Holding maturity constant, a bond's duration is higher when the coupon rate is lower.

Rule 3 Holding the coupon rate constant, a bond's duration generally increases with its time to maturity.

Rule 4 Holding other factors constant, the duration of a coupon bond is higher when the bond's yield to maturity is lower.

Rule 5 The duration of a level perpetuity is equal to: $(1+y) / y$

Bond Duration versus Bond Maturity



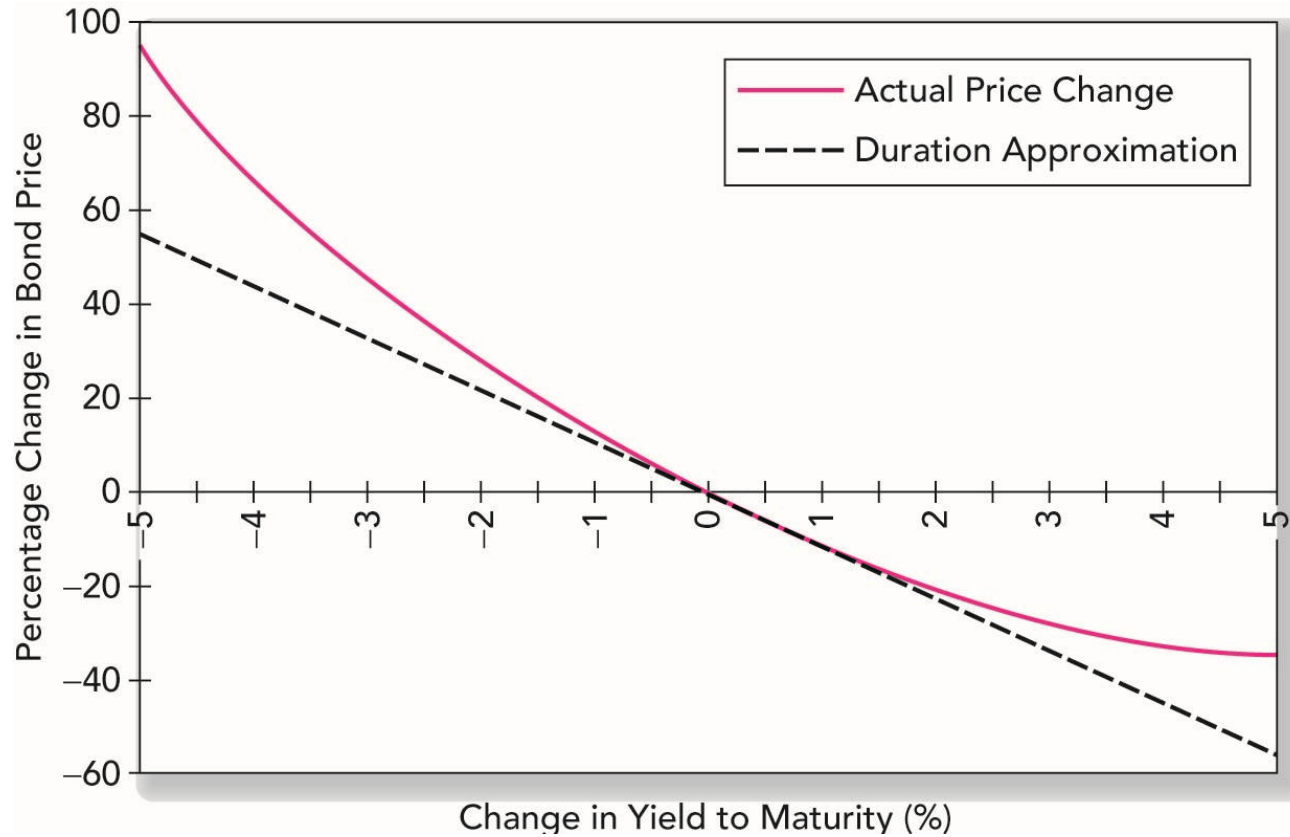
Source: BKM (2007)

Bond Duration (Initial Bond Yield 8% APR)

Years to Maturity	Coupon Rates (per Year)			
	6%	8%	10%	12%
1	0.985	0.980	0.976	0.972
5	4.361	4.218	4.095	3.990
10	7.454	7.067	6.772	6.541
20	10.922	10.292	9.870	9.568
Infinite (perpetuity)	13.000	13.000	13.000	13.000

Source: BKM (2007)

Bond Price Convexity (30-Year Maturity, 8% Coupon; Initial Yield to Maturity = 8%)



Source: BKM (2007)

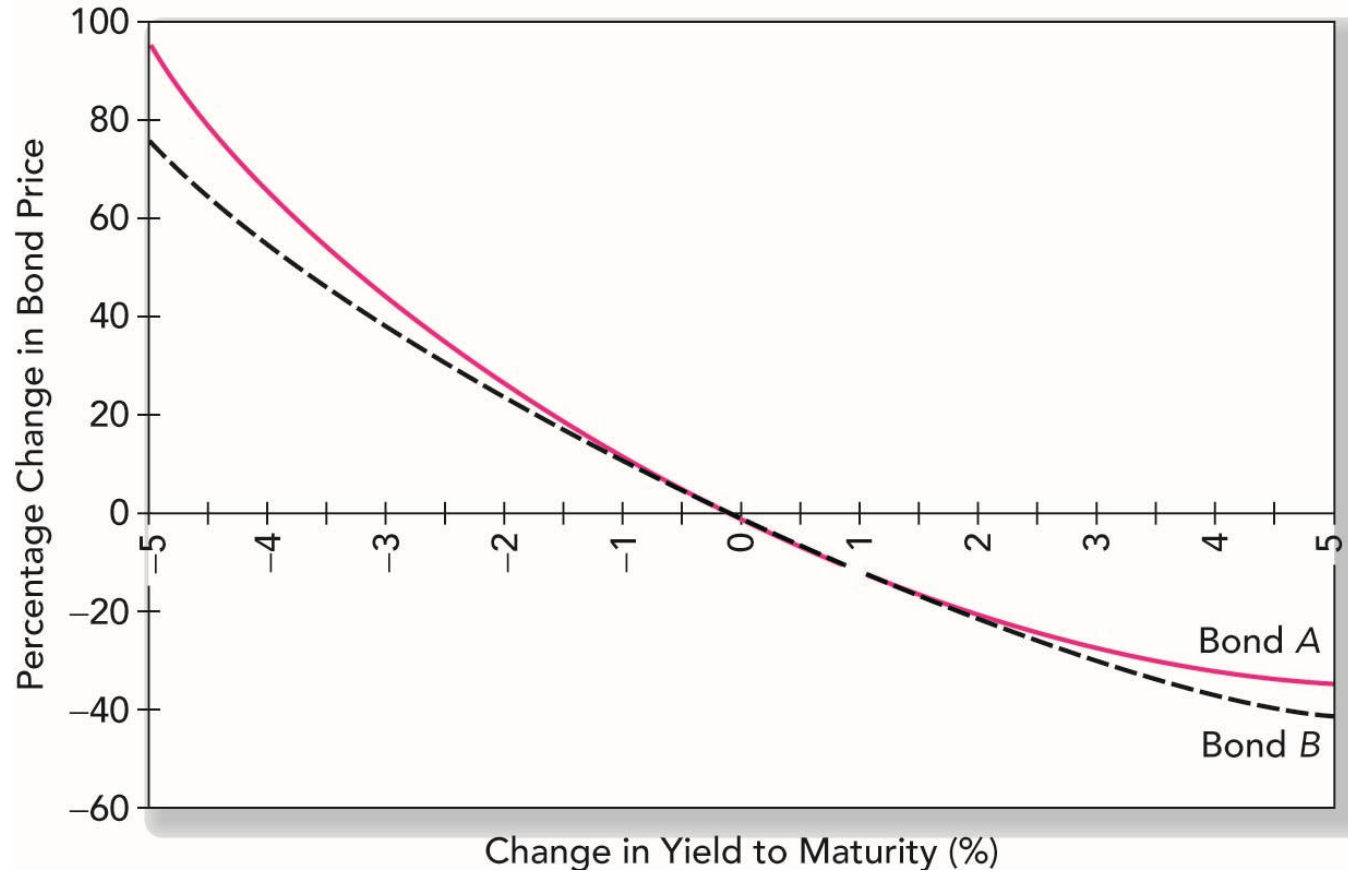
Correction for Convexity

$$\text{Convexity} = \frac{1}{P \times (1 + y)^2} \sum_{t=1}^n \left[\frac{CF_t}{(1 + y)^t} (t^2 + t) \right]$$

Correction for Convexity:

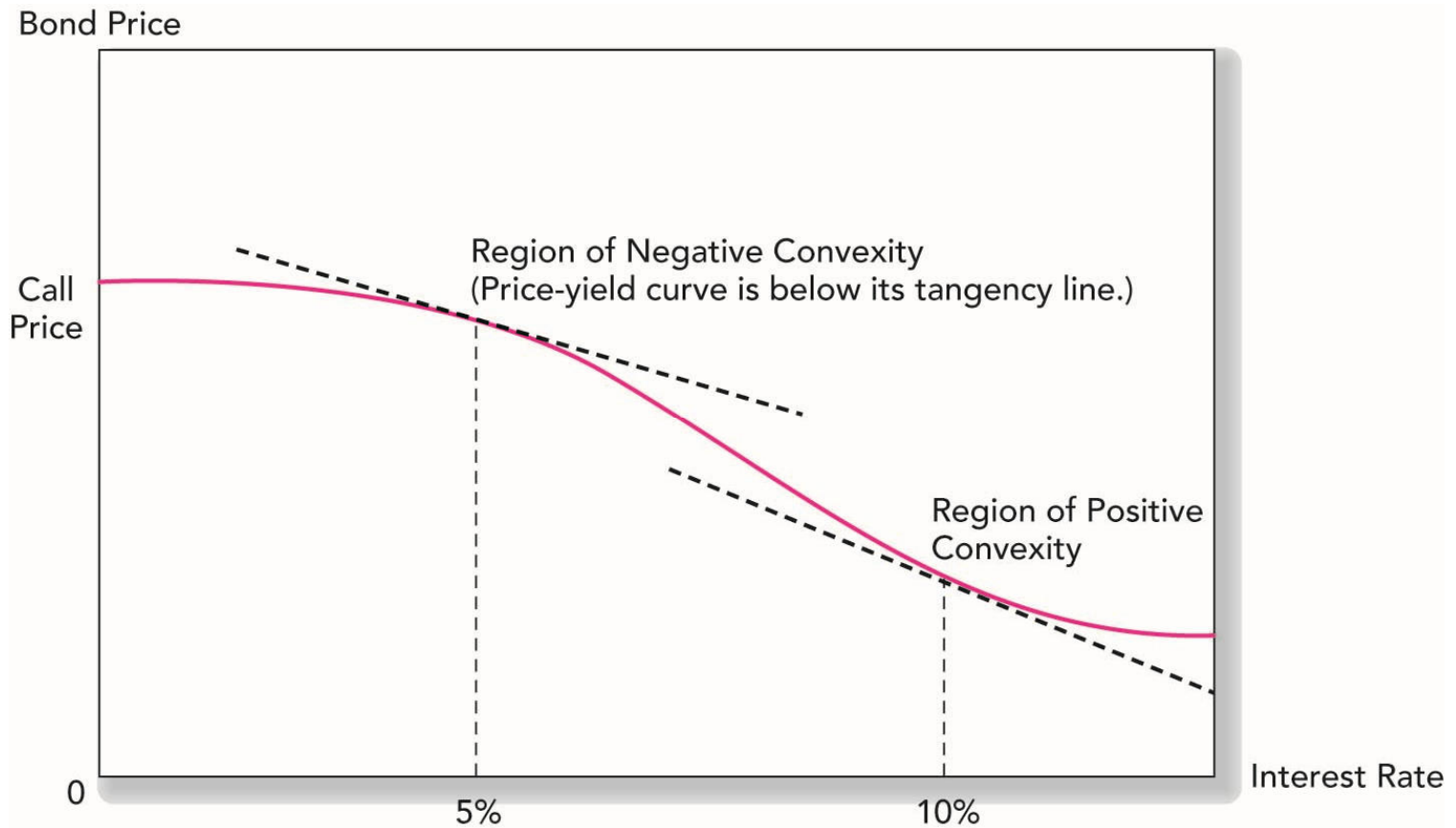
$$\frac{\Delta P}{P} = -D * \Delta y + \frac{1}{2} [\text{Convexity} \times (\Delta y)^2]$$

Convexity of Two Bonds



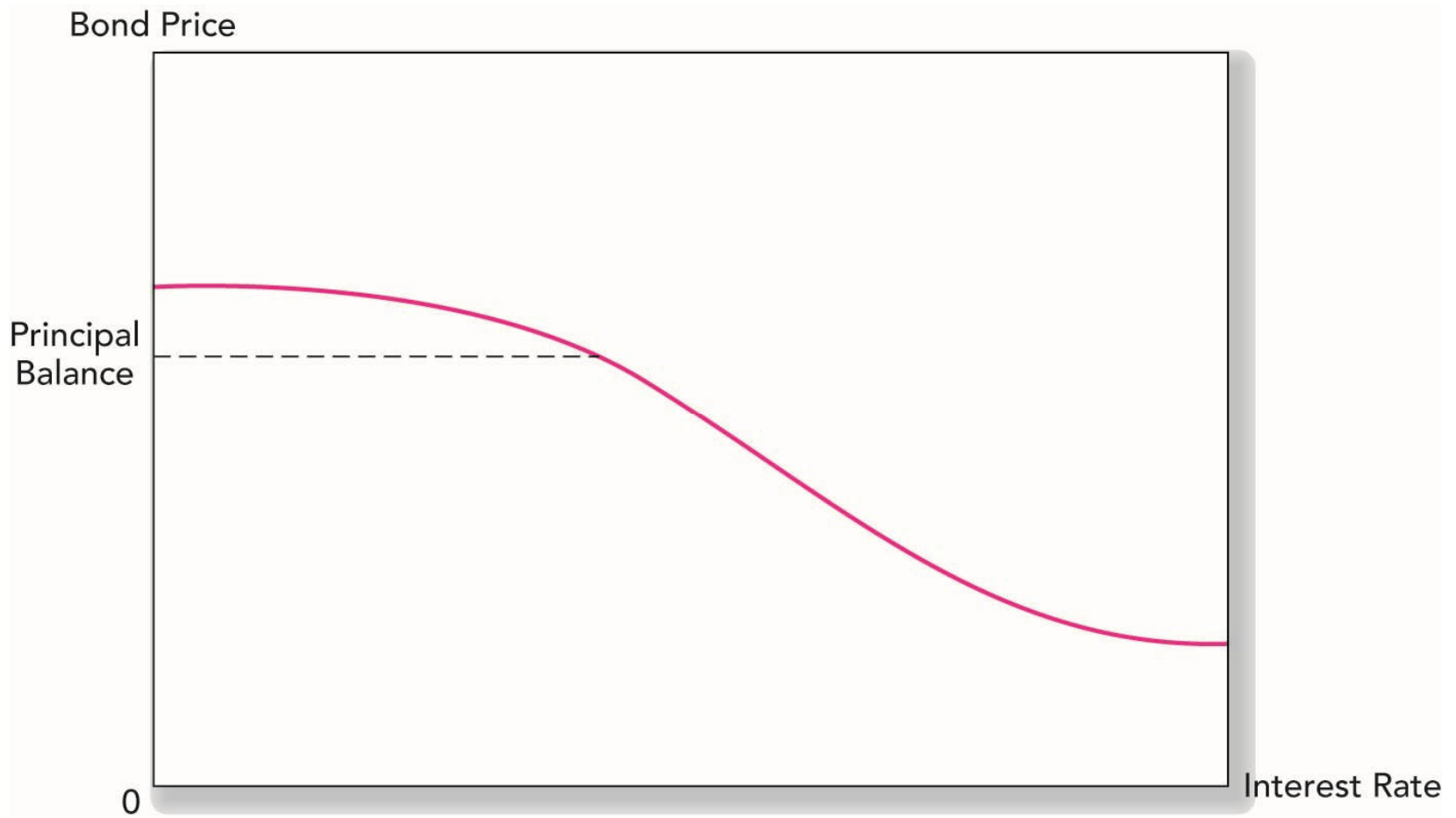
Source: BKM (2007)

Price –Yield Curve for a Callable Bond



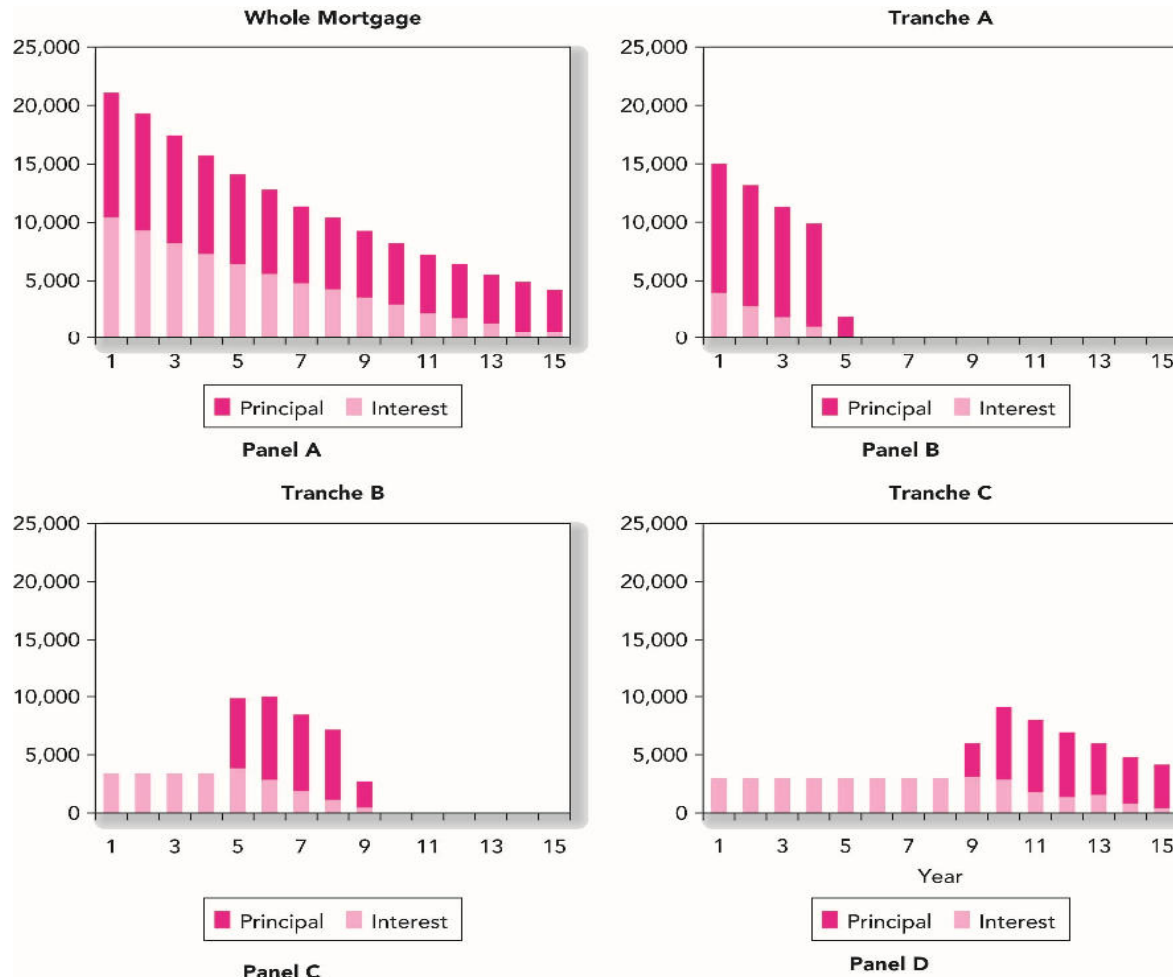
Source: BKM (2007)

Price – Yield Curve for a Mortgage-Backed Security



Source: BKM (2007)

Panel A: Cash Flows to Whole Mortgage Pool; Panels B – D Cash Flows to Three Tranches



Source: BKM (2007)

Passive Management

- Bond-Index Funds
- Immunization of interest rate risk:
 - Net worth immunization
 - Duration of assets = Duration of liabilities
 - Target date immunization
 - Holding Period matches Duration
- Cash flow matching and dedication

Stratification of Bonds into Cells

Term to Maturity \ Sector	Treasury	Agency	Mortgage-backed	Industrial	Finance	Utility	Yankee
<1 year	12.1%						
1-3 years	5.4%						
3-5 years			4.1%				
5-7 years							
7-10 years		0.1%					
10-15 years							
15-30 years			9.2%			3.4%	
30+ years							

Source: BKM (2007)

Terminal value of a Bond Portfolio After 5 Years (All Proceeds Reinvested)

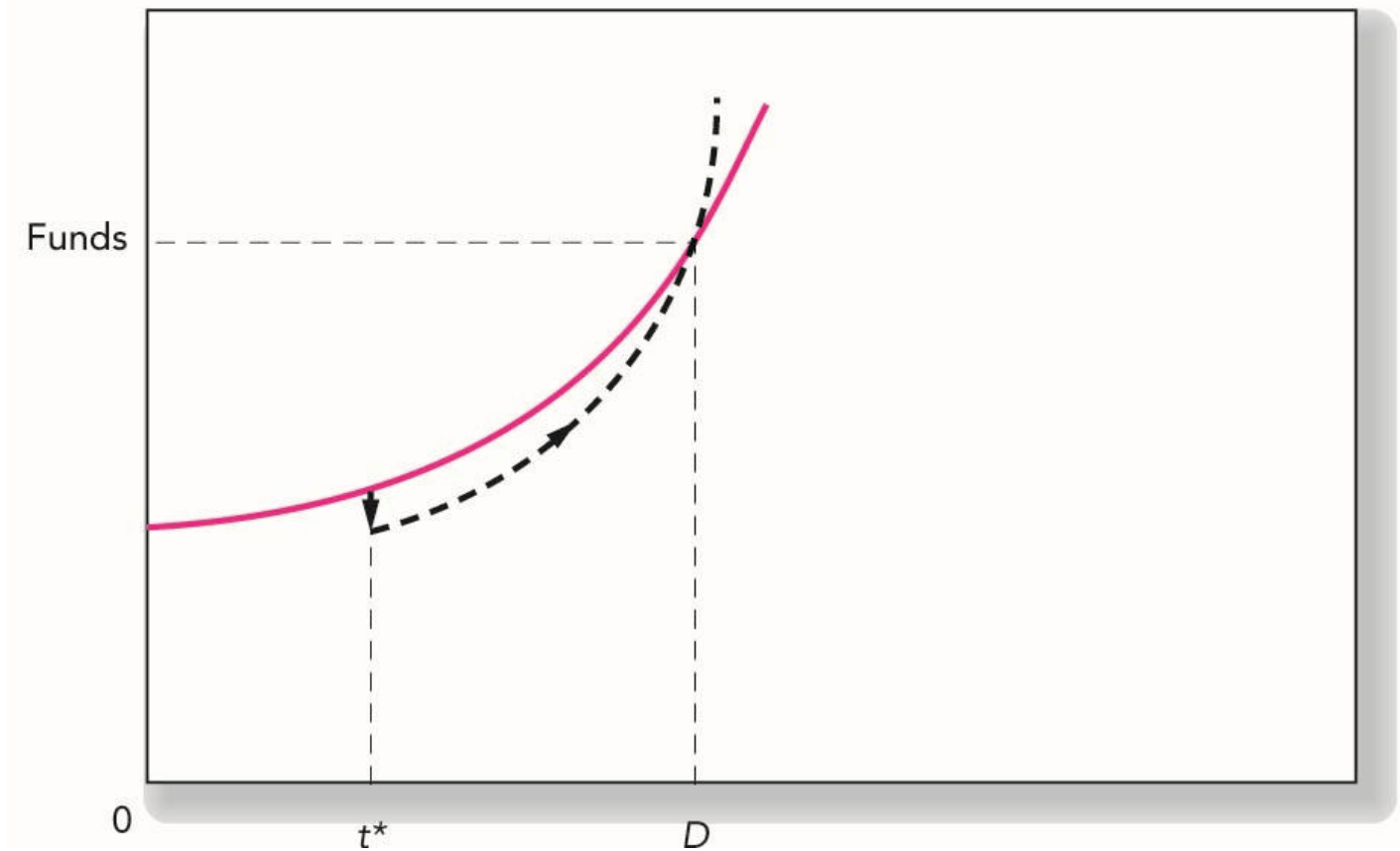
Payment Number	Years Remaining until Obligation	Accumulated Value of Invested Payment		
A. Rates remain at 8%				
1	4	$800 \times (1.08)^4$	=	1,088.39
2	3	$800 \times (1.08)^3$	=	1,007.77
3	2	$800 \times (1.08)^2$	=	933.12
4	1	$800 \times (1.08)^1$	=	864.00
5	0	$800 \times (1.08)^0$	=	800.00
Sale of bond	0	$10,800/1.08$	=	10,000.00
				<u>14,693.28</u>
B. Rates fall to 7%				
1	4	$800 \times (1.07)^4$	=	1,048.64
2	3	$800 \times (1.07)^3$	=	980.03
3	2	$800 \times (1.07)^2$	=	915.92
4	1	$800 \times (1.07)^1$	=	856.00
5	0	$800 \times (1.07)^0$	=	800.00
Sale of bond	0	$10,800/1.07$	=	10,093.46
				<u>14,694.05</u>
C. Rates increase to 9%				
1	4	$800 \times (1.09)^4$	=	1,129.27
2	3	$800 \times (1.09)^3$	=	1,036.02
3	2	$800 \times (1.09)^2$	=	950.48
4	1	$800 \times (1.09)^1$	=	872.00
5	0	$800 \times (1.09)^0$	=	800.00
Sale of bond	0	$10,800/1.09$	=	9,908.26
				<u>14,696.02</u>

Note: The sale price of the bond portfolio equals the portfolio's final payment (\$10,800) divided by $1 + r$, because the time to maturity of the bonds will be 1 year at the time of sale.

Source: BKM (2007)

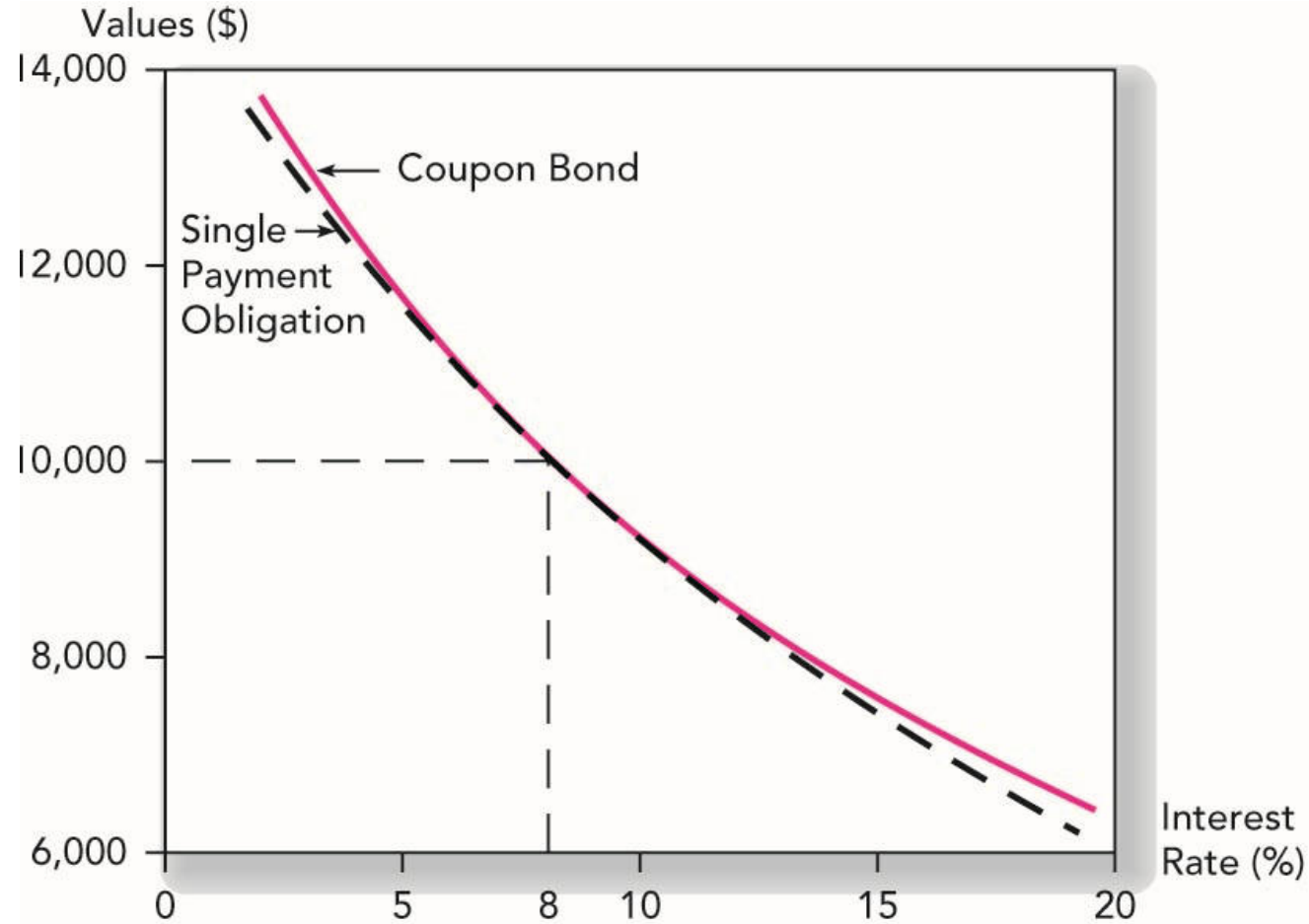
Growth of invested Funds

Accumulated Value of Invested Funds



Source: BKM (2007)

Immunization



Source: BKM (2007)

Market Value of Balance Sheet

Assets		Liabilities	
A. Interest rate = 8%			
Bonds	\$10,000	Obligation	\$10,000
B. Interest rate = 7%			
Bonds	\$10,476.65	Obligation	\$10,476.11
C. Interest rate = 9%			
Bonds	\$9,551.41	Obligation	\$ 9,549.62

Notes:

Value of bonds = $800 \times \text{Annuity factor}(r, 6) + 10,000 \times \text{PV factor}(r, 6)$

$$\text{Value of obligation} = \frac{14,693.28}{(1+r)^5} = 14,693.28 \times \text{PV factor}(r, 5)$$

Source: BKM (2007)

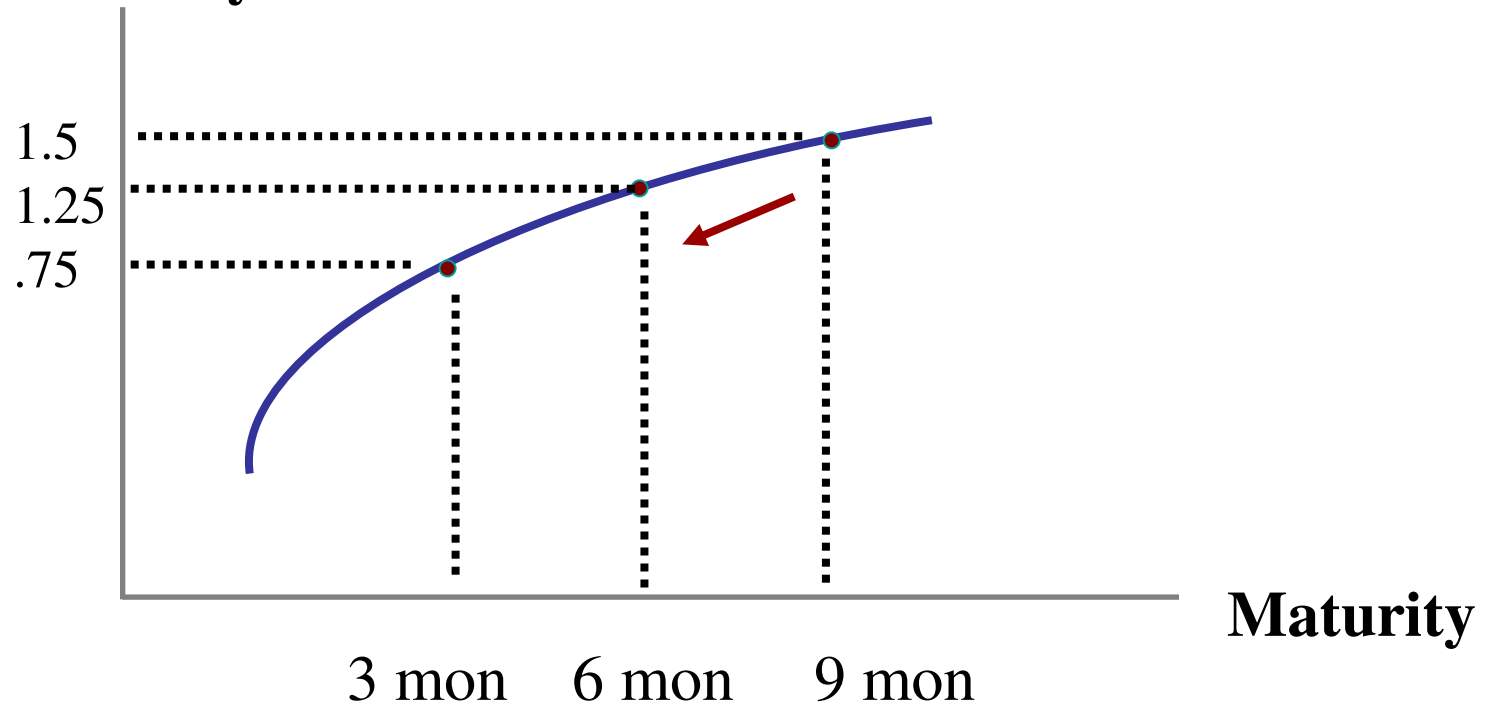


Active Management: Swapping Strategies

- Substitution swap
- Intermarket swap
- Rate anticipation swap
- Pure yield pickup
- Tax swap

Yield Curve Ride

Yield to
Maturity %

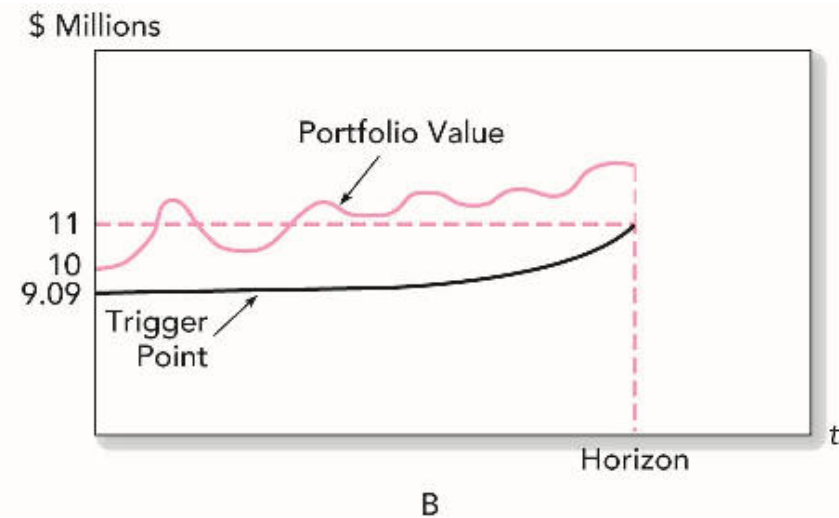
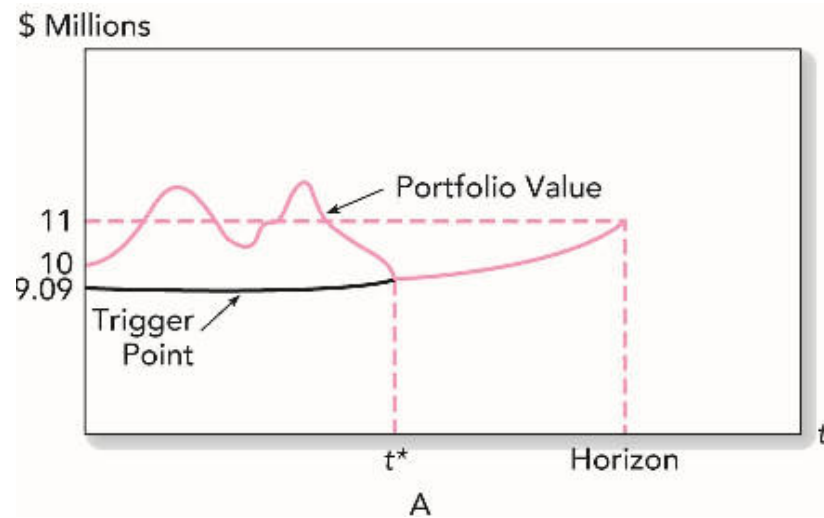


Source: BKM (2007)

Contingent Immunization

- A combination of active and passive management.
- The strategy involves active management with a floor rate of return.
- As long as the rate earned exceeds the floor, the portfolio is actively managed.
- Once the floor rate or trigger rate is reached, the portfolio is immunized.

Contingent Immunization



Source: BKM (2007)

Questions and Problems (1)

1. Given the following, does the law of one price hold? If not, what action should an investor take?

Bond	Cash-flow in period		
	1	2	3
A	100	1100	970
B	80	1080	936
C	90	1090	980

Questions and Problems (2)

2. Assume a bond with cash flows of \$100 each year and a principal payment of \$1000 in five years and a current price of \$960. What is
 - A. Its current yield
 - B. Its yield to maturity

3. Given the cash flows shown below, does the law of one price hold? If not, what is the price of a bond C that will make it hold

	Cash-flow in period		
Bond	1	2	Price
A	80	1080	982
B	1100		880
C	120	1120	1010

Questions and Problems (3)

4. Consider a bond with semiannual coupon payments of \$50, a principal payment of \$1000 in 5 years, and a price of \$1000. Assume that the yield curve is flat 10%. What is the duration of the Bond ?

5. Given the following Bonds, construct three different portfolios, each with a duration of 9 years.

Bond	Duration (years)
A	5
B	10
C	12

Thank you for your attention...

See you soon